ƎNERATION AND DISPLAY OF CONJUGATE SURFACES USING COMPUTER GRAPHICS

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY

by

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to the
DEPARTMENT OF MECHANICAL ENGINEERING
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JUNE, 1982

Dedicated

to

my Father

CERTIFICATE

This is to certify that the work" Generation and Display of Conjugate Surfaces Using Computer Graphics" by K. Krishnamurthy has been carried out under my supervision and has not been submitted elsewhere for a degree.

JULY 2, 1982

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ACKNOWLEDGEMENTS

I am indebted to Dr. S.G. Dhande who introduced me to the areas of Computer-Aided-Design and Computer Graphics. My sincere thanks to him for his invaluable cooperation in every step of the progress of this Thesis. His untiring efforts and moral support have led to the successful completion of this work.

I thank my colleagues M/s. P.V. Ramana, K.P. Reddy, B.C. Rama Naidu, Radha Krishna who made my stay in IIT Kanpur a pleasant experience. I also thank my friends M/s P.B. Ramulu, N.V.S. Murthy for their valuable cooperation and help during the preparation of this Thesis.

I wish to record my appreciation and thanks for the excellent computer facilities provided by Computer Centre, IIT Kanpur for carrying out this work.

I am thankful to Mr. D.P. Saini for careful and efficient typing of this Thesis.

June, 1982 K. Krishnamurthy

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NOMENCLATURE

S	Distance moved by the rack tooth.
$\emptyset$	Angle turned by the gear blank for the distance S moved by rack tooth.
R	Pitch circle radius of pinion in Chapter 2. radius of cylinder in Chapter 3.
$\dot{\emptyset}$	Angular velocity of the pinion.
x_G, y_G, z_G	Global coordinate system which is fixed at the centre of the pinion gear.
x_1, y_1, z_1	Coordinate system which is moving with rack tooth.
x_2, y_2, z_2	Reference coordinate system which is rotating with the gear blank.
β	Helix angle.
α	Pressure angle.
N	Number of teeth.
2H	Height of the rack tooth.
2u	Distance along the slant height of the rack tooth in front view.
2v	Distance along the face width of the tooth in top view.
MOD	Module of gear.
x, y, z	Global coordinate system.
2l	Length of the cylinder.

r_i, r_o	Inner and outer radius of cylinder from outside axis.
NPOS	Number of divisions.
NDIV	Number of divisions.
NPHI	Number of divisions in ϕ range.
NU, NW	Number of curves along u and w directions in a surface.
NLU, NLW	Number of curves along u and w directions in a surface patch.

ABSTRACT

In the present work, the geometrical models for the generation of conjugate surfaces have been developed. The two types of conjugate surfaces, one about surfaces in spur and helical gearing and the other about special conjugate surfaces have been discussed. Details of analytical derivations and display procedures in each case have been worked out. Based on the models proposed in the present work appropriate computer programs were developed. Using these programs some numerical examples have been worked out.

Chapter 1

INTRODUCTION

1.1 Classification of Surfaces :

The path of a moving point may be described by a position vector $\bar{r}$ at successive instants of time. Mathematically we can represent this space curve as

$$\bar{r} = \bar{r}\,(t) \quad \text{where } t \text{ represents time}$$

This is equivalent to

$$x = x\,(t)$$
$$y = y\,(t)$$
$$z = z\,(t) \quad \text{in terms of coordinates.}$$

Similarly, the space curve can be represented by a single scalar or parameter u . Mathematically,

$$\bar{r} = \bar{r}\,(u) \quad \text{represents a time independent space curve.}$$

Strictly speaking a surface is a set of two independent families of space curves as shown in Fig. 1.1. Any vector function $\bar{r} = \bar{r}\,(u, v)$ of two variables will represent a surface.

Hence,

$$x = x\,(u, v)$$
$$y = y\,(u, v)$$
$$z = z\,(u, v) \quad \text{represents the surface in terms of coordinates.}$$

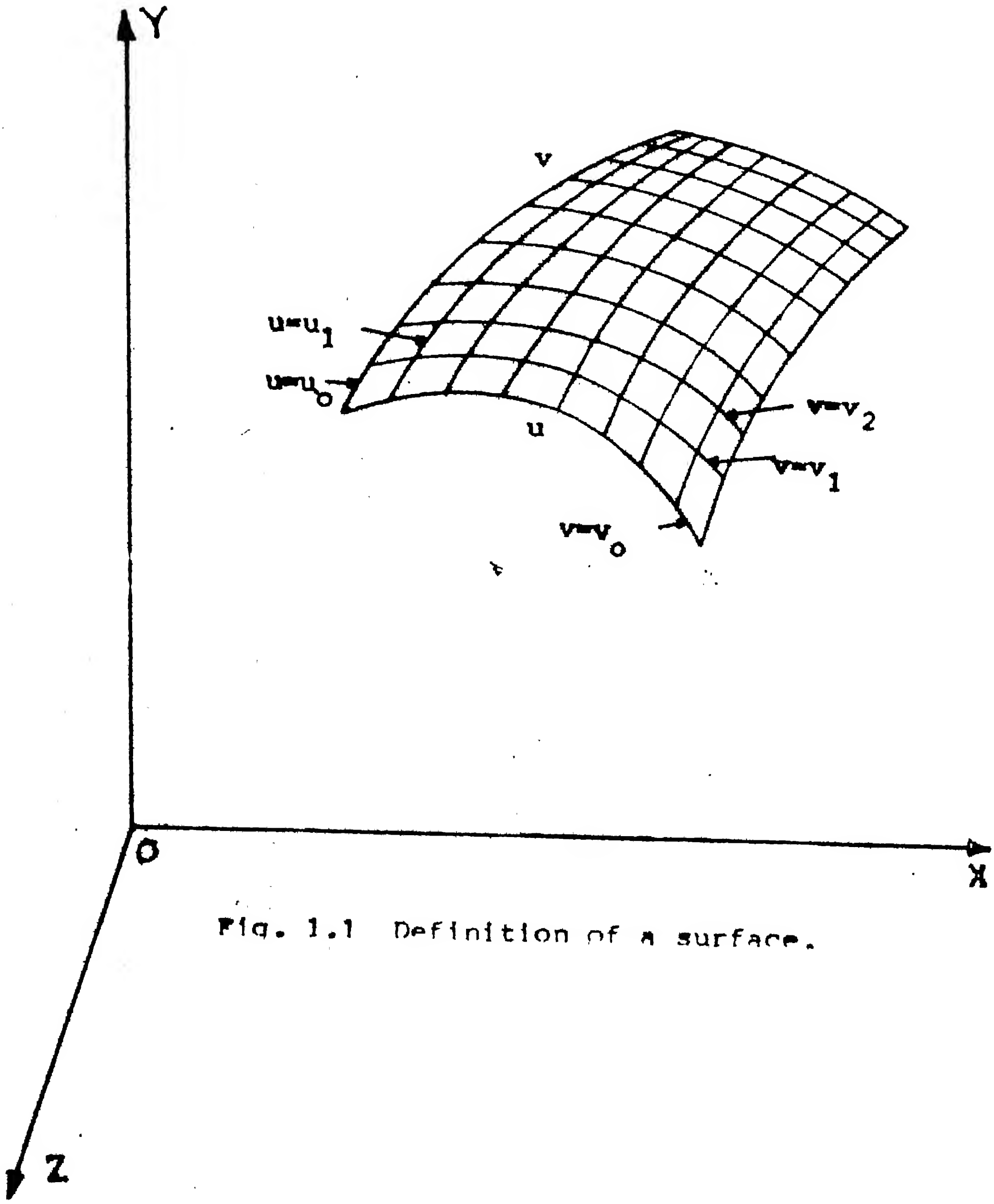

Fig. 1.1 Definition of a surface.

If either u or v is fixed, it will become a single variable parameter, so that vector equations $\bar{r} = \bar{r}\ (u_o, v)$ or $\bar{r} = \bar{r}\ (u, v_o)$ represent curves lying on the surface-provided that u_o and v_o are constants in the range of u and v which describe the surface. Such curves are called parametric curves on the surface [11].

There are four classes of surfaces [1] :

1. Plane surfaces
2. Single curved surfaces
3. Double curved surfaces
4. Warped surfaces.

1.1.1 <u>Plane Surfaces</u> :

A plane surface is generated by a straight line which moves so as to touch two intersecting straight lines for example Prisms, Pyramids.

1.1.2 <u>Single Curved Surfaces</u> :

A single curved surface is generated by the motion of a straight line which touches a curve and in which any two consecutive elements are parallel or intersecting. These will have nonzero curvature in only one direction, for example cylinders, cones.

1.1.3 Double Curved Surfaces :

This can be generated by moving a curve, or a variable curve, so as to produce a surface which is not ruled. The methods of generation include revolving a curve about an axis, moving a curve along another curve and varying a curve while it is moving. These are classified as (a) Double curved surfaces of revolution and (b) All other double curved surfaces.

1.1.4 Surfaces of Revolution :

The revolution of any line about an axis generates a surface of revolution. The surface produced may be single curved, double curved or warped.

1.1.4a Single Curved Surfaces of Revolution :

These are generated by revolving a straight line about an axis that is in the same plane. If the line and the axis are parallel, the surface is a cylinder of revolution. If they intersect the surface is a cone of revolution.

1.1.4b A Warped Surface of Revolution :

This is generated by revolving a straight line about an axis that is not in the same plane.

1.1.4c <u>Double Curved Surfaces of Revolution</u> :

The revolution of any plane curve about an axis generates a double curved surface of revolution. The <u>Sphere</u> is generated by the revolution of a circle about one of its diameters. The <u>Annular Torus</u> is generated by the revolution of a circle about an axis outside the circle. The <u>Ellipsoids</u> of revolution will be generated by revolving a ellipse about either axis. A <u>Paraboloid</u> of revolution is generated by revolving a parabola about its axis. The <u>Hyperboloid</u> is generated by the revolution of hyperbola about its transverse axis.

1.1.5 <u>Warped Surfaces</u> :

This is generated by the continuous motion of a straight line such that no two consecutive positions lie in the same plane.

1.1.5a <u>The Parabolic Hyperboloid</u>:

This is generated by the continuous motion of a straight line (generatrix) which touches two skew lines (directrices) and remains parallel to a plane director. The generatrix is the moving straight line which generates the surface. The directrix is straight or curved line which the generatrix continuously touches. The director is a surface to which the generatrix remains parallel.

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1.1.5b The Cylindroid :

This is generated by the continuous motion of a straight line which touches two plane curves and remains parallel to a plane director.

1.1.5c The Conoid :

This is generated by the continuous motion of a straight line which touches a circle,an ellipse, or a plane curve and a straight line and remains parallel to a plane director. If the straight line is perpendicular to the plane director, the surface is a right conoid. If it is inclined the surface is an oblique conoid.

1.1.5d The Helicoid :

This is generated by the continuous motion of a straight line which touches a helix and its axis and remains parallel to a plane perpendicular to the axis of helix. If it is not perpendicular to the axis of helix, it is called oblique helicoid.

1.2 Methods of Generating Surfaces :

1.2.1 Introduction :

Surfaces of any required shape can be produced in machine tools, employing various cutting processes and suitable cutting tools. The surfaces can be produced by a combination of rotary and rectilinear uniform motions of the cutting tool and work [6].

The surfaces which are classified in Section 1.1 are regarded as the trace produced in the motion of one generating line - the generatrix, along another line - the directrix. Surfaces may be enveloping, or internal, or external, or closed, or open. Work pieces that are to be machined rarely have only one surface. In such cases, the whole surface is made up of a series of elementary surfaces. For instance, a spur gear can be conceived as two sets of identical cylindrical surfaces (with an involute directrix), arranged symmetrically about the gear axis constituting the right and left sides of a tooth. The top land of each tooth is an open surface of a circular cylinder while the surface at the bottom of the tooth is also an open cylindrical surface with a directrix of some specified form.

The geometrical shape of most engineering surfaces can be shaped by using generating lines of the following types:

A. Lines realized in machine tools by means of simple - rotary and rectilinear and only uniform motions, such as : (1) straight lines (2) circles or their arcs (3) involutes of circles (4) helixes on cylinders, cones etc. (5) spirals (6) epicycloids (7) hypocycloids (8) space curves produced by uniform rotary and rectilinear motions.

B. Lines realized in machine tools by means of both uniform and non-uniform simple, rotary and rectilinear motions such as:(1) parabolas (2) hyperbolas (3) ellipses (4) sine curves and (5) logarithmic spirals.

It is by means of these lines that the shape of a surface is specified. For example, the side surface of a helical gear tooth is specified by the form of its generating lines.. It will be an involute cylindrical helicoid.

1.2.2 Shaping Geometrical and Real Surfaces :

Real surfaces, obtained on a solid body by any method of working the material (casting, press working, machining) are approximation of the corresponding geometrical surfaces. A geometrical surface is defined as the trace obtained in the motion of one geometrical generating line called the generatrix along another geometrical line called the directrix. Real surfaces can be shaped on metal or other materials with the aid of auxiliary real surfaces, lines and points which are referred to as auxiliary material elements. The geometrical generating lines and consequently, the required surfaces are produced in the motion of these real auxiliary elements. The relative motions of the geometrical lines in producing surfaces

are shown in Fig. 1.2.

1.2.2a The Forming Method (Fig. 1.2a) :

In this method the configuration and extent or length, of the auxiliary material line coincides with the configuration and extent of the line being produced. The geometrical line is produced without any formative motions.

1.2.2b The generating Method (Fig. 1.2b) :

In this method the line is obtained as the envelope of the consecutive positions occupied by the auxiliary element (in the form of a line) as it rolls along the line being produced. Here the formative motion is rolling.

1.2.2c The Tracing Method (Fig. 1.2c) :

In this, the auxiliary element in the form of a material point produces the line being formed as the trace as it leaves in its motion (The material point may be the short length of the cutting edge on the tool).

1.2.2d The Tangent Method (Fig. 1.2d) :

In this method, the line being formed is tangent to a series of supplementary auxiliary lines, produced by the material point either by the tracing method or the tangent method.

To produce a specified surface, it is necessary to have a geometrical generatirx and directrix of corresponding configuration which can be formed by any one of

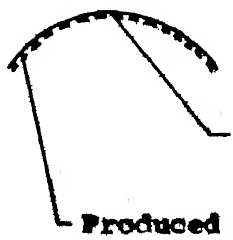


Fig. 1.2a

Fig. 1.2b

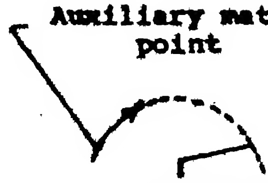


Fig. 1.2c

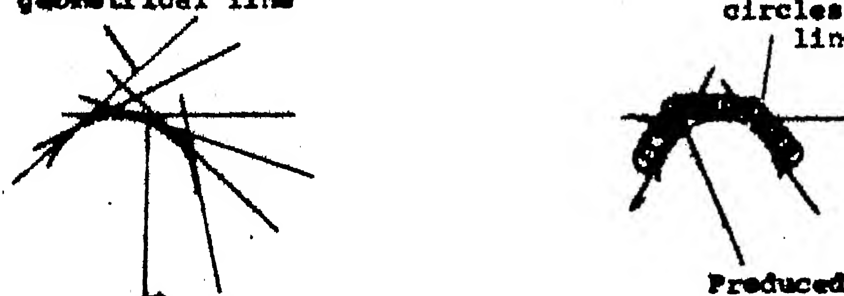


Fig. 1.2d

the four methods discussed above. Several geometrical methods of shaping surfaces have been described, in detail, by Acherkan and others [6].

1.3 Theory of conjugate Surfaces :

Consider two surfaces (Fig. 1.3) Σ_1 and Σ_2 attached to links 1 and 2 respectively. Let the links 1 and 2 be moving links and their motion is described by parameters p_1 and p_2 respectively. In general the parameters represent linear motion or angular motion. Let at the given instant the positions of links 1 and 2 be represented by coordinate systems S_1 $(X_1\ Y_1\ Z_1 - O_1)$ and S_2 $(X_2\ Y_2\ Z_2 - O_2)$. In the initial position these coordinate systems coincide with a set of coordinate systems $S_{\bar{1}}$ and $S_{\bar{2}}$ which are fixed in space. Let S (X, Y, Z - O) represent a global coordinate system fixed in space. If during continuous motion of links 1 and 2 the surfaces are in contact with one another transmitting required motion as described by parameters p_1 and p_2, then the surfaces are called as the conjugate surfaces. These conjugate surfaces are mutually enveloping surfaces generated due to the motion of links 1 and 2.

The condition that the two surfaces are conjugate and transmit required motion has been given by Dhande and Chakraborty [7], Litvin [8], Dyson [9] and others.

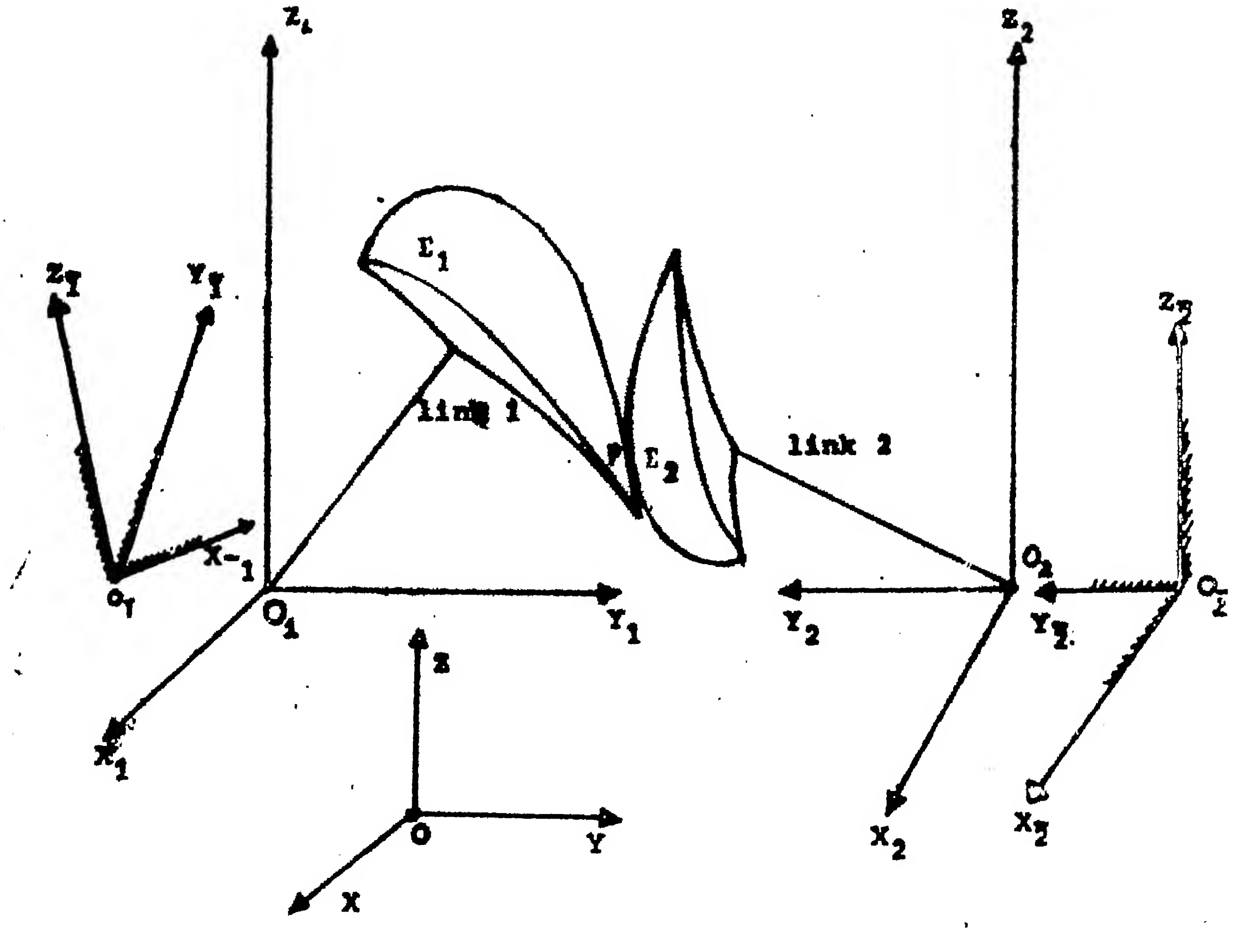


Fig. 1.3 Definition of a conjugate surface.

The equations that are useful for further work are given below.

Let $\bar{r}_1^{(p)}$ represent the radius vector of a generic point p lying on the surface Σ_1 of link 1 and $\bar{n}_1^{(p)}$ is the unit normal vector at p to the surface Σ_1. $\bar{r}_2^{(p)}$, $\bar{n}_2^{(p)}$ are the similar quantities for the surface Σ_2. If p is the point of contact then the following conditions are satisfied.

$$[M_{01}]\,\bar{r}_1^{(p)} = [M_{02}]\,\bar{r}_2^{(p)} \qquad (1.1)$$

$$[L_{01}]\,\bar{n}_1^{(p)} = [L_{02}]\,\bar{n}_2^{(p)} \qquad (1.2)$$

where

$$\bar{r}_1^{(p)} = [\, x_1\ (u, v)\ ,\ y_1\ (u, v),\ z_1\ (u, v),\ 1]^T$$

$$\bar{n}_1^{(p)} = [\, n_{1x}(u, v)\ ,\ n_{1y}(u, v),\ n_{1z}(u, v)\ ]^T$$

$$\bar{n}_1^{(p)} = \frac{\dfrac{\partial \bar{r}_1}{\partial u} \times \dfrac{\partial \bar{r}_1}{\partial v}}{\left|\dfrac{\partial \bar{r}_1}{\partial u} \times \dfrac{\partial \bar{r}_1}{\partial v}\right|}$$

and $\bar{r}_2^{(p)}$ and $\bar{n}_2^{(p)}$ are supposed be unknowns of a conjugate surface. The condition of conjugacy states that at the point of contact the common normal should be orthogonal to the relative velocity vector between the two

links at p so at p

$$\bar{n}^{(1)} \cdot \bar{v}^{(12)} = 0 \qquad (1.3)$$

where

$$\bar{n}^{(1)} = [L_{01}] \; \bar{n}_1^{(p)}$$

$$\bar{v}^{(12)} = \bar{v}^{(1)} - \bar{v}^{(2)}$$

$$= \frac{d}{dt} [M_{01}] \; \bar{r}_1^{(p)} - \frac{d}{dt} [M_{02}^{-1}] \, [M_{01}] \; \bar{r}_1^{(p)}$$

Equation (1.3) is a scalar equation in which input parameters are the motion parameters p_1 and p_2. So for a given value of p_1 and p_2, Equation (1.3) can be solved for u and v of surface Σ_1. Once u and v are found out, then that locates point p on Σ_1 and Σ_2. The conjugate profiles can be found out from Equation (1.1).

1.4 Scope of the Present Work :

Aim of the present work is to develop appropriate procedures for computer-aided graphical design and display of conjugate surfaces. Given the surface generating machine (Milling machine, Lathe, Grinder, Drilling machine, Shaper etc.) and also the characteristics of the machine (input and output parameters), one can develop the same surface on the computer feeding these machine characteristics as the input of the program developed. For instance,

machining of turbine blades, propeller blades which are in curved shape is difficult. Knowing the parameters Ø, H, S one can machine this type of surface. In another case the hyperbolic paraboloid surface will be developed by choosing Ø, H, S appropriately. Limitations of the present work is that one has to know the characteristics of surface generating machine which unfortunately change with the type of the machine. Hence the characteristic parameters of the machine are not fed as the input of the program developed. In Chapter 2 the surfaces involved in spur and helical gearing will be discussed. Special conjugate surfaces will be discussed in Chapter 3. In section 1, the generation of hyperbolic paraboloid surface, in section 2, the generation of right helicoid, in section 3, the generation of oblique helicoid will be discussed in Chapter 3.

Chapter 4 will discuss the computational considerations. In section 1 of this chapter the multiview display arrangement, in section 2, the surface generation technique, in section 3, the flow charts for the programs developed, in section 4,the illustrative examples will be discussed.

In Chapter 5 conclusions about the work that has been done will be discussed. In section 1, the technical summary and in section 2, recommendations for future work will be discussed.

Chapter 2

CONJUGATE GEAR PROFILES

2.1 Surfaces in Spur Gearing :

2.1.1 About Spur Gears :

When the rack tooth is moving a distance S, the gear blank rotates through an angle ϕ about its centre. Considering the point p on the rack surface as shown in Fig. 2.1 the point of contact p traces the path of straight line as the rack is moving rectilinearly. The point p generates the involute profile (which is conjugate to the rack profile) in $x_2\ y_2\ z_2\text{-}O_2$ coordinate system attached to gear blank. The involute surface generation is given below.

2.1.2 Involute Surface Generation :

As shown in Fig. 2.1, if u and v are the parameters describing the location of a generic point p on the rack surface, then

$$\bar{r}_1^{(p)} = [\ -\ u \sin\alpha \qquad u \cos\alpha \qquad v \quad 1]^T \qquad (2.1)$$

$$\bar{n}_1^{(p)} = [\ \cos\alpha \qquad \sin\alpha \qquad 0]^T$$

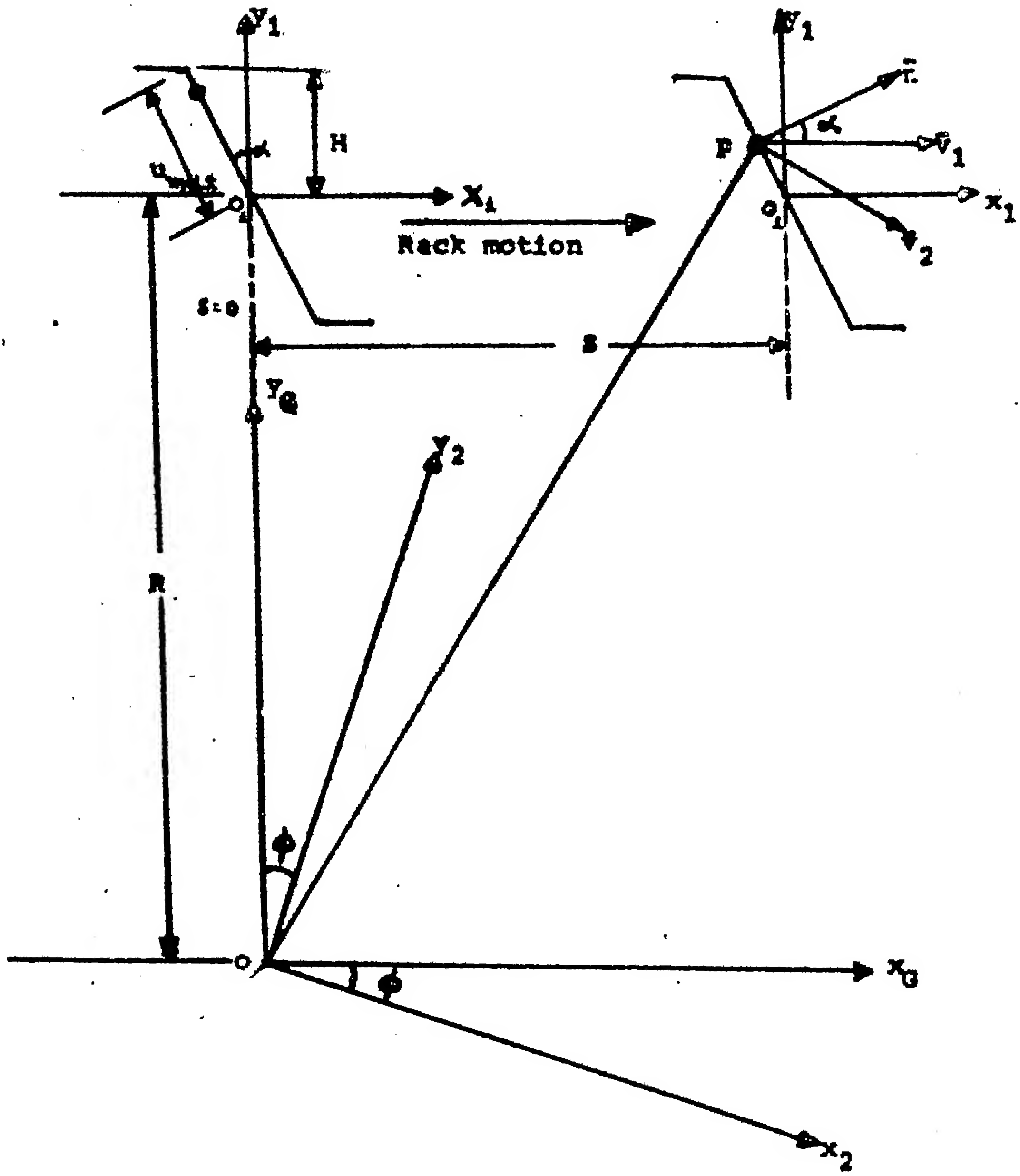


Fig. 2.1 Generation of spur gear involute surface.

Moreover, the radius vector and the unit normal vector can be expressed with reference to the global coordinate system $x_G y_G z_G - O_G$ as follows:

$$\bar{r}^{(p)} = \begin{bmatrix} S - u\sin\alpha \\ R + u\cos\alpha \\ v \\ 1 \end{bmatrix}, \qquad \bar{n}^{(p)} = \begin{bmatrix} \cos\alpha \\ \sin\alpha \\ 0 \end{bmatrix} \tag{2.2}$$

The relative velocity vector $\bar{v}_r^{(12)}$ at point p when expressed in the global coordinate system is given by

$$\bar{v}_r^{(12)} = \begin{bmatrix} \dot{S} - (R + u\cos\alpha)\,\dot{\phi} \\ (S - u\sin\alpha)\,\dot{\phi} \\ 0 \end{bmatrix} = \dot{\phi}\begin{bmatrix} -u\sin\alpha \\ (S - u\sin\alpha) \\ 0 \end{bmatrix} \tag{2.3}$$

As discussed in section 1.3, the relative velocity vector and the common normal vector are mutually perpendicular at the point of contact. Therefore, using Equations (2.2) and (2.3),

$$\bar{v}_r^{(12)} \cdot \bar{n}^{(p)} = 0$$

Substituting proper expressions,

$$u = S\sin\alpha \tag{2.4}$$

The gear profile is given by,

$$\bar{r}_2^{(p)} = [M_{20}]\ \bar{r}^{(p)}$$

(i.e.) $x_2 = (S - u\sin\alpha)\cos\phi - (R + u\cos\alpha)\sin\phi$

$$y_2 = (S - u\sin\alpha)\sin\phi + (R + u\cos\alpha)\cos\phi \quad (2.5)$$

$$z_2 = v$$

From the above set of equations one can plot the x_2, y_2, z_2 coordinates to get the involute surface for various values of z_2 along the face width of the rack tooth as well as gear tooth.

2.1.3 Calculation of Maximum and Minimum Movement of Rack Tooth :

From Fig. 2.2

$$AB^2 + (2R\sin\alpha)AB + (R^2 - TCR^2) = 0$$

$$AB = \frac{-(2R\sin\alpha) \pm \sqrt{(2R\sin\alpha)^2 - 4(R^2 - TCR^2)}}{2.0},$$

TCR is top circle radius

$$S_{max} = AM = AB/\cos\alpha \quad (2.6)$$

$$S_{min} = AR = -(MOD/\cos\alpha)\frac{1}{\sin\alpha}, \text{ MOD is module}$$

2.1.4 Algorithm :

1. Read α, N, NPOS, MOD, NLU, NLW as input data.
2. Calculate S_{max}, S_{min} values with the given gear data using Eq (2.6).
3. Compute S_i, i = 1, NPOS $\quad S_{min} \leq S_i \leq S_{max}$
4. Compute v_K, K = 1, NPOS ; $-$ Face width/2.0 $\leq v_K \leq$ Face width/2.0.

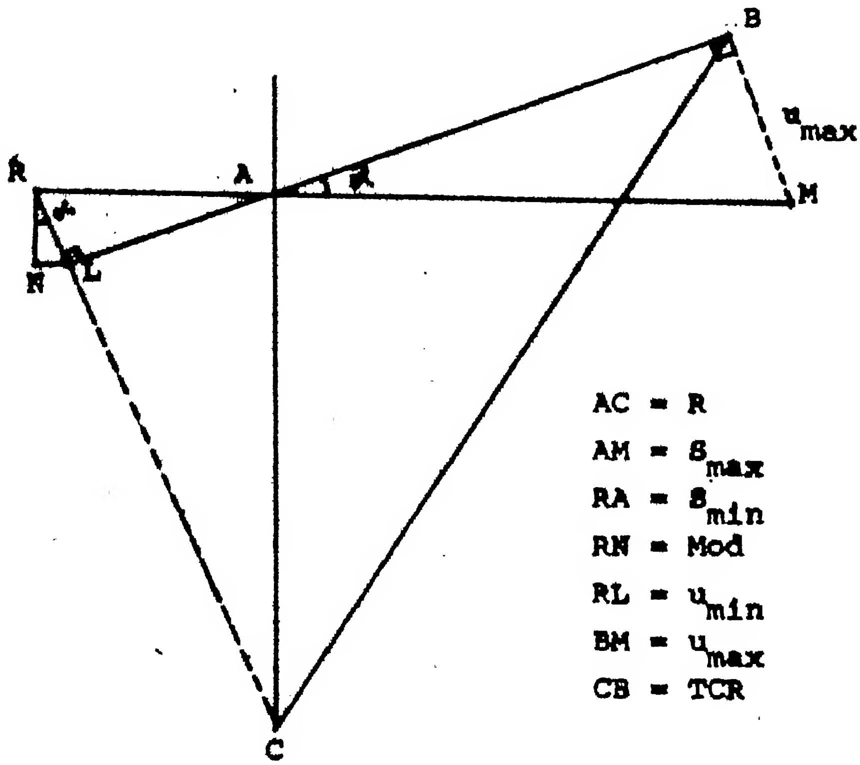


Fig. 2.2 Calculation of S_{max} and S_{min} values.

5. Compute ϕ_i , u_i $i = 1,NPOS$ using $\phi_i = S_i/R$ and Eq. (2.4)

6. Compute x_G, y_G, z_G, x_1 , y_1, z_1, x_2, y_2 for a fixed value of v_K. varying ϕ_i, u_i $i = 1, NPOS$ using Eq. (2.2), (2.1) and (2.5).

7. For v_K $K = 1, NPOS$ repeat the step 6.

8. With the x_1, y_1, z_1 coordinates generate one side of the rack tooth face, use the mirror reflection of the same surface and extensions according to standard gear data [2], complete the rack tooth.

9. With the x_2, y_2, z_2 coordinates generate one side of the gear tooth using coons surface program. Complete gear tooth using this surface [2].

2.2 Surfaces in Helical Gearing :

2.2.1 About Helical Gears :

The theory of developing the conjugate involute surface is same as that discussed in section 2.1.1 except the formulae. The formulae are given in section 2.2.2. Helical gears transmit motion between parallel shafts. The shape of the tooth is involute helicoid. The initial contact of helical gear tooth is a point which changes into a line as the tooth come into more engagement. In spur

gears the line of contact is parallel to the axis of rotation, where as in helical gears the line is diagonal across the face of the tooth. Because of this gradual engagement of the tooth, these gears transmit heavy loads at high speeds.

2.2.2 Generation of Involute Surface :

If u and v are the parameters describing the location of a generic point p on the rack surface, then

$$\bar{r}_1^{(p)} = [(-v \sin\beta - u \sin\alpha \cos\beta), u \cos\alpha, (v \cos\beta - u \sin\alpha \sin\beta), 1]^T \quad (2.7)$$

$$\bar{n}_1^{(p)} = [\cos\alpha \cos\beta \quad \sin\alpha \quad \cos\alpha \sin\beta]$$

Moreover, the radius vector and the unit normal vector can be expressed with reference to the global coordinate system $x_G\ y_G\ z_G - O_G$ as follows :

$$\bar{r}^{(p)} = \begin{bmatrix} (S - u \sin\alpha \cos\beta - v \sin\beta) \\ (R + u \cos\alpha) \\ (v \cos\beta - u \sin\alpha \sin\beta) \\ 1 \end{bmatrix} \qquad \bar{n}^{(p)} = \begin{bmatrix} \cos\alpha \cos\beta \\ \sin\alpha \\ \cos\alpha \sin\beta \end{bmatrix} \quad \ldots \quad (2.8)$$

The relative velocity vector $\bar{v}_r^{(12)}$ at point p when expressed in the global coordinate system is given by

$$\bar{v}_r^{(12)} = \begin{bmatrix} -u\cos\alpha\,\dot{\phi} \\ (S - v\sin\beta - u\sin\alpha\cos\beta)\dot{\phi} \\ 0 \end{bmatrix} = \dot{\phi}\begin{bmatrix} -u\cos\alpha \\ (S - v\sin\beta - u\sin\alpha\cos\beta) \\ 0 \end{bmatrix} \qquad \ldots \quad (2.9)$$

At the point of conjugacy,

$$\bar{v}_r^{(12)} \cdot \bar{n}^{(p)} = 0$$

Substituting appropriate expressions,

$$u = \frac{(S - v\sin\beta)\sin\alpha}{\cos\beta} \qquad (2.10)$$

The gear profile is given by

$$\bar{r}_2^{(p)} = [M_{20}]\ \bar{r}^{(p)}$$

(i.e.)

$$x_2 = (S - v\sin\beta - u\sin\alpha\cos\beta)\cos\phi - (R + u\cos\alpha)\sin\phi$$

$$y_2 = (S - v\sin\beta - u\sin\alpha\cos\beta)\sin\phi + (R + u\cos\alpha)\cos\phi$$

$$z_2 = (v\cos\beta - u\sin\alpha\sin\beta) \qquad \ldots. \quad (2.11)$$

From Eq (2.11) one can generate involute surface with x_2, y_2, z_2 coordinates using coons surface program for various values of z_2 along the face width of the rack tooth as well as gear tooth. S_{max}, S_{min} can be calculated as discussed in section 2.1.3.

2.2.3 Algorithm :

1. Read N, α, β, NPO^S, NLU, NLW as input data.
2. Compute S_i, i = 1, NPOS $\quad S_{min} \leq S_i \leq S_{max}$

3. Compute $\phi_i = S_i/R \quad i = 1, NPOS$

4. Compute $v_K \quad K = 1, NPOS$ along the face width of rack tooth.

5. Compute u_i, $i = 1, NPOS$ using Eq (2.10).

6. For $i = 1, NPOS$ compute x_1, y_1, z_1 and x_2, y_2, z_2 values using Eq (2.7) and (2.11) varying u_i, S_i; ϕ_i for a fixed value of v_K.

7. Repeat the step 6 for $K = 1, NPOS$ and compute all the discrete coordinate values.

8. Generate one side of the rack tooth with x_1, y_1 z_1 values using coons program.

9. Generate one side of the gear tooth with x_2, y_2, z_2 values using coons program.

2.2.4 <u>Helical Gear Tooth Generation</u> :

The involute surface generated above with the lines of contact between rack tooth and pinion tooth is not the actual one side of the tooth surface. The lines should be clipped such that the lines lie inside the tooth surface. The clipping is done as follows

$$BCR \leq \sqrt{x_2^2 + y_2^2} \leq TCR \tag{2.12}$$

where BCR, TCR are base circle radius and top circle radius respectively. Discard the points which are not satisfying Eq (2.12). The remained points will develop

the one side of the tooth face. Using the standard gear data [2] complete the helical gear tooth. Similarly, for rack tooth clip the lines such that the points satisfy

$$- H \leqslant \sqrt{x_1^2 + y_1^2} \leqslant H \qquad (2.13)$$

The points which are satisfying Eq (2.13) will generate one side of the rack rooth. The rack tooth will be completed using standard gear data [2].

Chapter 3

SPECIAL CONJUGATE SURFACES

3.1 Generation of Hyperbolic Paraboloid :

3.1.1 Definition :

The hyperbolic paraboloid is warped surface generated by the continuous motion of a straight line (generatrix) which touches two skew lines (directrices) and remains parallel to the plane director.

3.1.2 Derivations :

As shown in Fig. 3.1, the cylinder is moving with $\dot{S}$ linear velocity in the z_1 direction and at the same time it is precessing about z_1 axis. S is the distance moved by cylinder in z_1 direction. $x_1 y_1 z_1 - O_1$ coordinate system is fixed to the cylinder, $x_I y_I z_I - O_I$ is the coordinate system which is at S distance from the initial position of the cylinder. xyz - O is the global coordinate system fixed in space.

$S = \phi h$ where h is a proportionality constant.

$\dot{S} = \dot{\phi} h$

where $\dot{S}$ is the linear velocity of cylinder in z_1 direction.

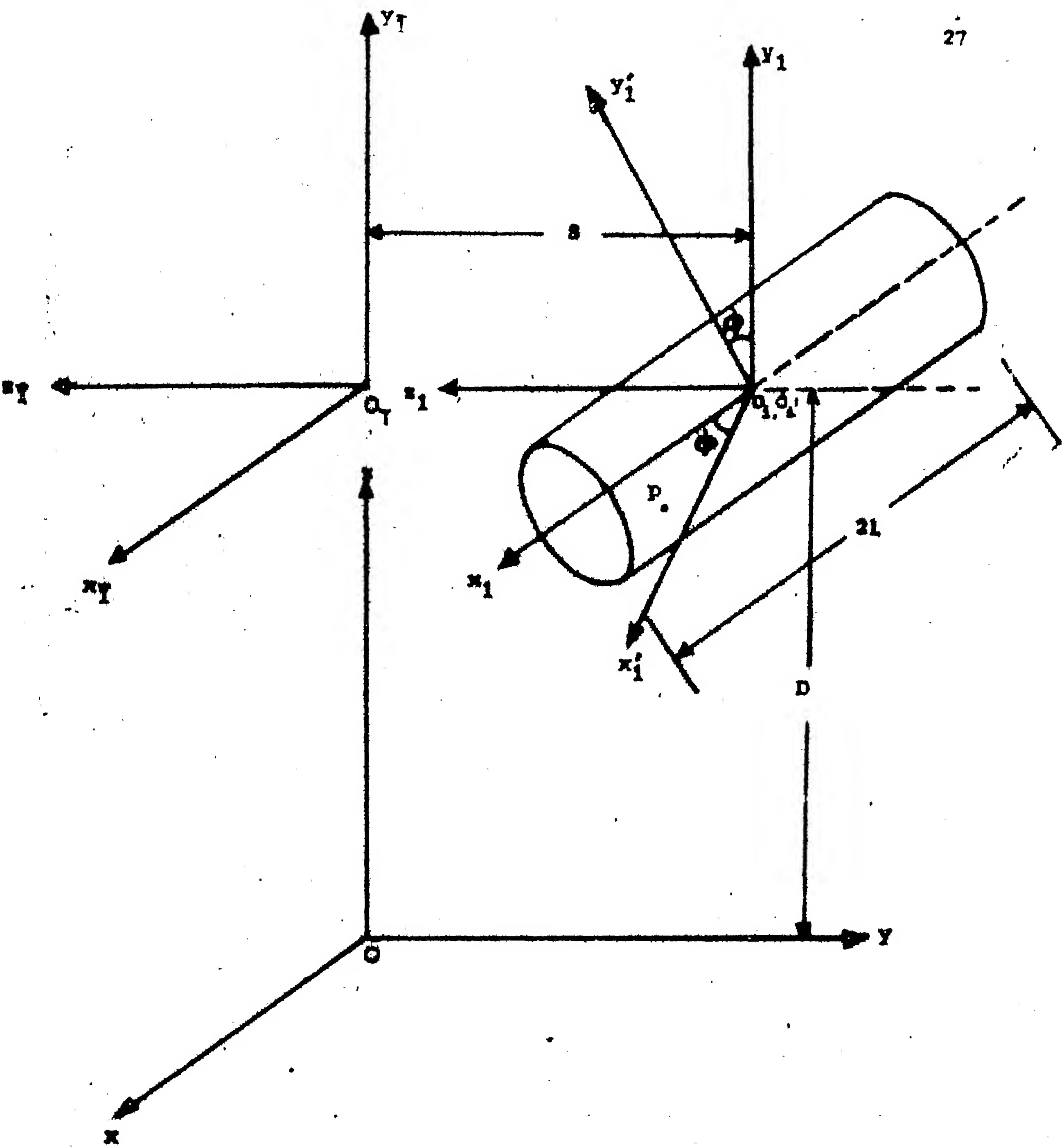


Fig. 3.1 Generation of a hyperbolic paraboloid.

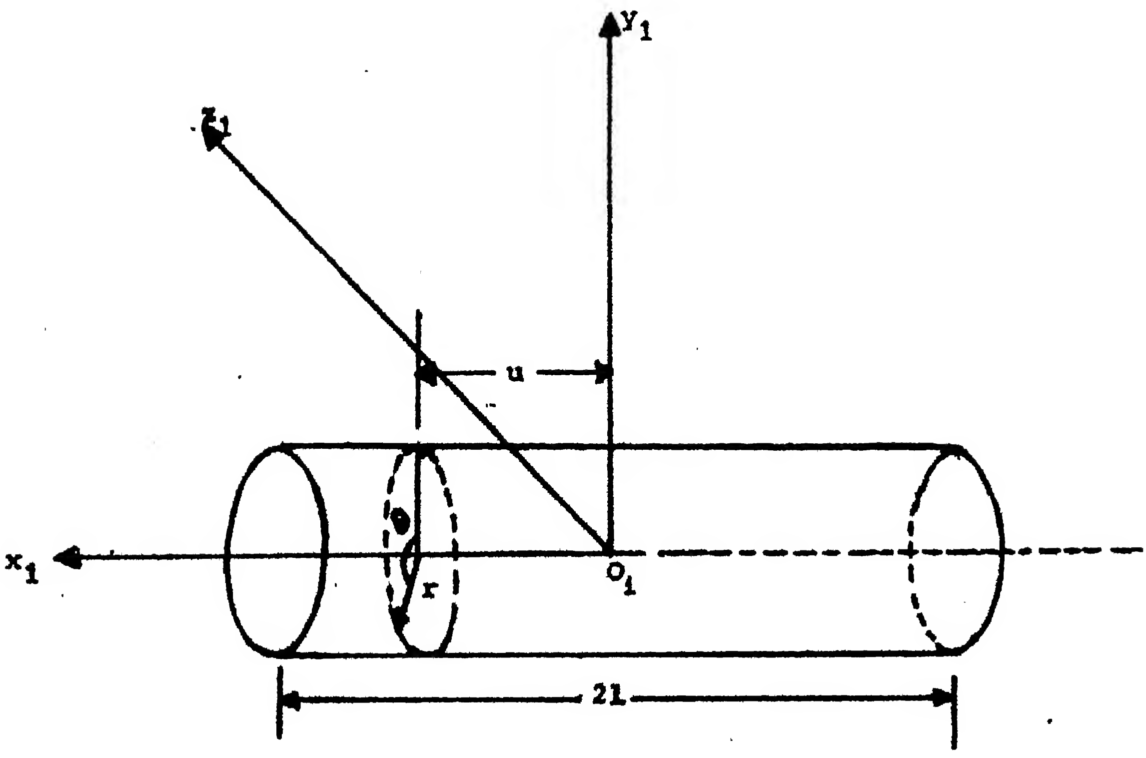


Fig. 3.2 Details of a rotating cylinder.

$\dot{\phi}$ is the angular velocity of cylinder about z_1 axis.

$$S_{min} \leqslant S \leqslant S_{max}$$

$$\phi_{min} \leqslant \phi \leqslant \phi_{max}.$$

The position vector of point p in $x_1\ y_1\ z_1 - O_1$ coordinate system can be expressed in $x_{\bar{1}}\ y_{\bar{1}}\ z_{\bar{1}} - O_{\bar{1}}$ coordinate system as ,

$$\begin{bmatrix} x_{\bar{1}} \\ y_{\bar{1}} \\ z_{\bar{1}} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 & 0 \\ \sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & -S \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} \quad (3.3)$$

The point p can be expressed with respect to the global coordinate system xyz - O as follows.

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & D \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{\bar{1}} \\ y_{\bar{1}} \\ z_{\bar{1}} \\ 1 \end{bmatrix} \quad (3.4)$$

From (3.3) and (3.4) we get,

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 & 0 \\ 0 & 0 & -1 & S \\ \sin\phi & \cos\phi & 0 & D \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} \quad (3.5)$$

The relative velicity vector $\bar{v}_r^{(12)}$ at point p when expressed in global coordinate system xyz - O is given by

$$\bar{v}_r^{(12)} = \dot{\phi} \begin{bmatrix} -\sin\phi & -\cos\phi & 0 & 0 \\ 0 & 0 & -1 & H \\ \cos\phi & -\sin\phi & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -x_1 \times \sin\phi - y_1 \times \cos\phi \\ H \\ x_1 \times \cos\phi - y_1 \times \sin\phi \end{bmatrix} \dot{\phi} \qquad (3.6)$$

From Fig. 3.2

$$\begin{aligned} x_1 &= u \\ y_1 &= r\cos\theta \\ z_1 &= r\sin\theta \end{aligned} \qquad (3.7)$$

Unit normal vector at point p when expressed in global coordinate system is given by

$$\bar{n} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ 0 & 0 & -1 \\ \sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \cos\theta \\ \sin\theta \end{bmatrix} = \begin{bmatrix} -\sin\phi\cos\theta \\ -\sin\theta \\ \cos\phi\cos\theta \end{bmatrix} \qquad (3.8)$$

At the point of conjugacy,

$$\bar{n} \cdot \bar{v}_r^{(12)} = 0 \qquad (3.9)$$

From (3.6), (3.7) , (3.8) and (3.9) we get,

$$\theta = \tan^{-1} (u/H) \text{ or } \pi + \tan^{-1} (u/H) \qquad (3.10)$$

From (3.5) and (3.7) we get,

$$\begin{aligned} x &= u \cos\phi - r \cos\theta \sin\phi \\ y &= - r \sin\theta + \phi H \qquad (3.11) \\ z &= u \sin\phi + r \cos\theta \cos\phi + D \end{aligned}$$

Equations (3.11) represent the equations of hyperbolic paraboloid surface.

$$- 1 \leqslant u \leqslant + 1$$

$$\phi_{min} \leqslant \phi \leqslant \phi_{max}$$

3.1.3 Algorithm :

1. Read l, H, R, D, NPOS, NPHI as input data.
2. Compute u_i $i = 1$, NPOS $- 1 \leqslant u_i \leqslant + 1$
3. Compute ϕ_K, $S_K = \phi_K h$ $K = 1$, NPHI
   $\phi_{min} \leqslant \phi_K \leqslant \phi_{max}$
4. For a fixed value of ϕ_K , compute x,y,z coordinates using eq (3.10) and (3.11) varying u_i, $i = 1$,NPOS
5. Repeat the step 4 for $K = 1$, NPHI
6. With NPOS X NPHI values of x,y,z coordinates generate the hyperbolic paraboloid surface using coons surface program.

3.1.4 Uses of Hyperbolic Paraboloid Surface :

This surface is found on some concrete walks and floors, other concrete work such as the wing walls for abutments may have this surface. This surface is being used by Architechts for roofs and other structures to obtain interesting effects. One of the most important applications of this surface is machining of the turbine blades, propeller blades etc. which is discussed in section 1.4.

3.2 Generation of Right Helicoid :

3.2.1 Definition :

The right helicoid is a warped surface generated by the continuous motion of a straight line which touches a helix and its axis and remains parallel to a plane perpendicular to the axis of the helix. The helix may be right or left hand. Another type of helicoid is one in which the elements are tangent to a coaxial cylinder instead of intersecting axis.

3.2.2 Derivations :

Right helicoid is obtained by rotating the cylinder about z axis as shown in Fig. 3.3 and the cylinder axis (x_1) is at right angles to the axis of rotation (z). Here both z and z_1 axes coincide with each other. The

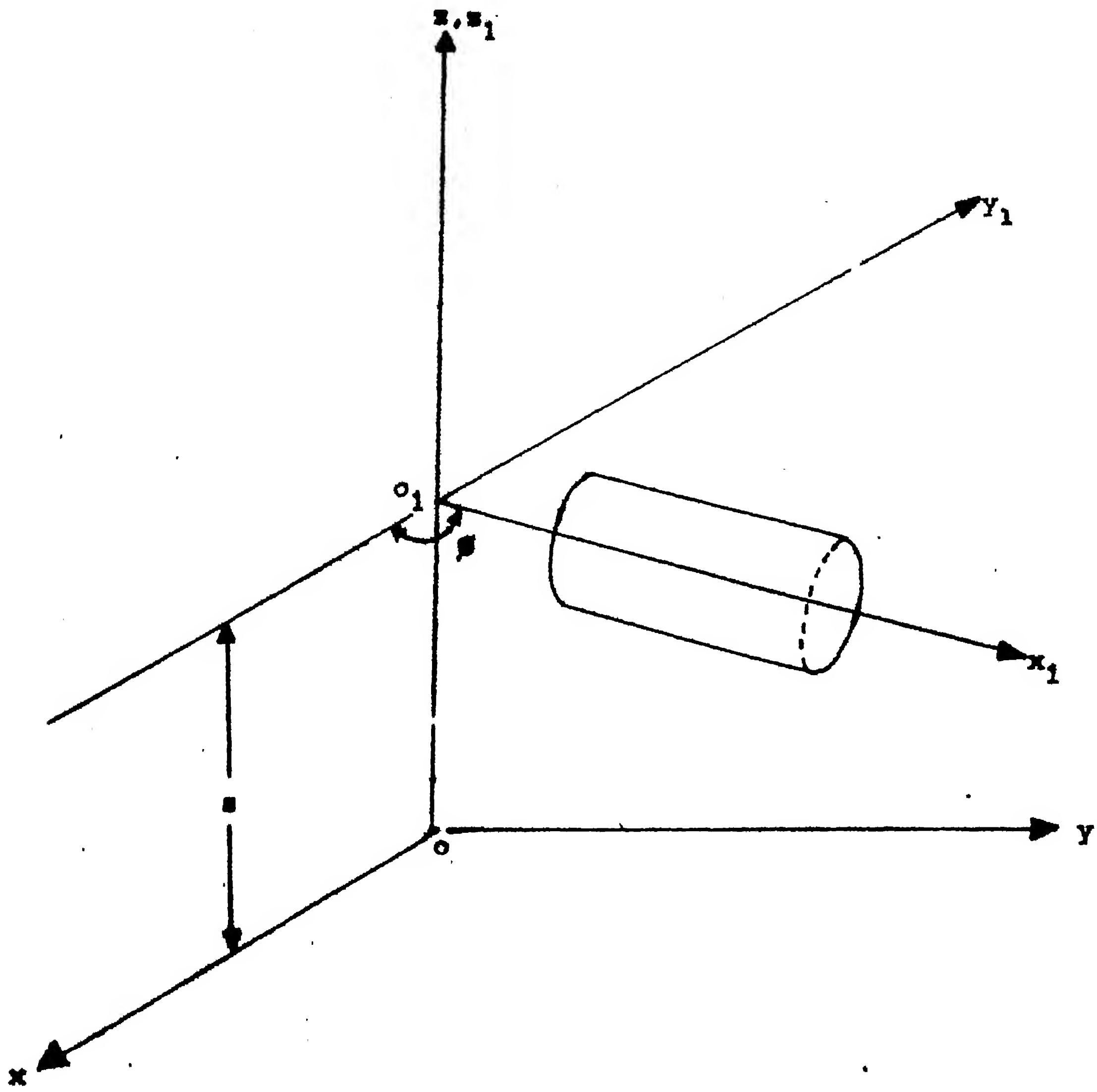


Fig. 3.3 Generation of a right helicoid.

cylinder rotates about its own axis (x_1) and also it rotates about another axis (z), at the same time the cylinder axis is moving along z axis. Then the right helicoid will be formed.

From Fig. 3.3

$$\begin{aligned} x_1 &= u \\ y_1 &= r\cos\theta \\ z_1 &= r\sin\theta \end{aligned} \qquad (3.12)$$

The position vector of point p can be expressed in global coordinate system (xyz - 0) as

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 & 0 \\ \sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & s \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} \qquad (3.13)$$

The unit normal vector at point p can be expressed in global coordinate system as

$$\bar{n} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \cos\theta \\ \sin\theta \end{bmatrix} = \begin{bmatrix} -\sin\phi\cos\theta \\ \cos\phi\cos\theta \\ \sin\theta \end{bmatrix} \qquad (3.14)$$

The relative velocity vector at point p can be expressed in global coordinate system as,

$$\bar{v}_r^{(12)} = \dot{\phi} \begin{bmatrix} -\sin\phi & -\cos\phi & 0 & 0 \\ \cos\phi & -\sin\phi & 0 & 0 \\ 0 & 0 & 0 & H \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} \quad (3.15)$$

At the point of conjugacy,

$$\bar{n} \cdot \bar{v}_r^{(12)} = 0 \quad (3.16)$$

From (3.12), (3.14), (3.15) and (3.16) we get,

$$\theta = \tan^{-1}(-u/H) \text{ or } \pi + \tan^{-1}(-u/H) \quad (3.17)$$

$$r_i \leqslant u \leqslant r_o$$

$$0 \leqslant \phi \leqslant 2\pi$$

From (3.12) and (3.13) we get,

$$\begin{aligned} x &= u\cos\phi - r\cos\theta\sin\phi \\ y &= u\sin\phi + r\cos\theta\cos\phi \\ z &= S + r\sin\theta \end{aligned} \quad (3.18)$$

Set of equations in (3.18) represent the right helicoid surface.

3.2.3 Algorithm :

1. Read r_i, r_o, R, H, D, NDIV, NPHI as input data.
2. Compute u_i $i = 1$, NDIV $r_i \leqslant u_i \leqslant r_o$
3. Compute ϕ_K $K = 1$, NPHI $0 \leqslant \phi \leqslant 2\pi$
4. Compute $S_K = \phi_K H$ $K = 1$, NPHI

5. For a fixed value of ϕ_K, compute x,y,z coordinates using eqs (3.17) and (3.18) , varying u_i i = 1,NDIV.
6. Repeat the step 5 for K = 1, NPHI
7. Generate the surface with NDIV X NPHI values of x,y,z coordinates using coons surface program.

3.2.4 Uses of Right Helicoid :

Right helocoids are used for screw conveyers. The working surfaces of square screw threads are right helicoids. Circular stairways are constructed on the basis of this surface.

3.3 Generation of Oblique Helicoid :

3.3.1 Definition :

The oblique helicoid is a warped surface generated by the continuous motion of a straight line which touches a helix and its axis and makes a constant angle, other than 90°, with the axis. The helix may be right hand or left hand. The helicoid may have elements which slope downward or upward from the helix.

3.3.2 Derivations :

The cylinder is rotating about its own axis x_2 which is at $(90 + \psi)$ degrees to the z-axis as shown in Fig. 3.4. z and z_2 axes are making ψ degrees each other. The cylinder is rotating about z-axis and simultaneously

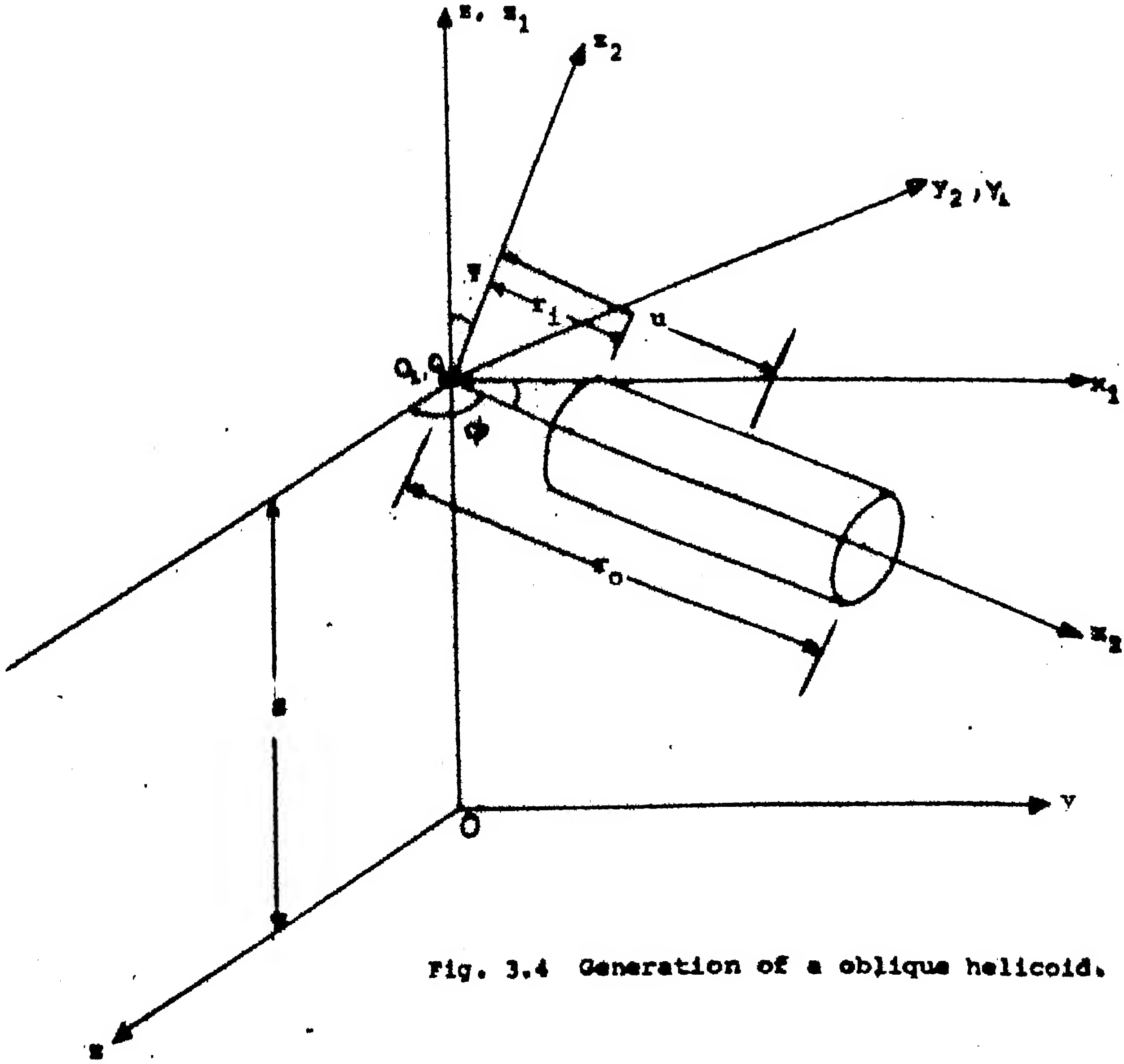


Fig. 3.4 Generation of a oblique helicoid.

$x_2\ y_2\ z_2 - O_2$ coordinate system is moving up or down along z axis.

For the configuration shown in Fig. 3.4, the point p can be expressed in global coordinate system (xyz - O) as

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} (\cos\phi\ \cos\Psi) & -\sin\phi & (\cos\phi\ \sin\Psi) & 0 \\ (\sin\phi\ \cos\Psi) & \cos\phi & (\sin\phi\ \sin\Psi) & 0 \\ \sin\Psi & 0 & \cos\Psi & S \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix} \qquad \ldots \quad (3.19)$$

$$x_2 = u$$
$$y_2 = r\cos\theta \qquad (3.20)$$
$$z_2 = r\sin\theta$$

Unit normal vector at point p can be expressed in global coordinate system as

$$\bar{n} = \begin{bmatrix} (\cos\phi\ \cos\Psi) & -\sin\phi & (\cos\phi\ \sin\Psi) \\ (\sin\phi\ \cos\Psi) & \cos\phi & (\sin\phi\ \sin\Psi) \\ -\sin\Psi & 0 & \cos\Psi \end{bmatrix} \begin{bmatrix} 0 \\ \cos\theta \\ \sin\theta \end{bmatrix}$$

$$\bar{n} = \begin{bmatrix} (-\sin\phi\ \cos\theta + \cos\phi\ \sin\Psi\ \sin\theta) \\ (\cos\phi\ \cos\theta + \sin\phi\ \sin\theta\ \sin\Psi) \\ \sin\theta\ \cos\Psi \end{bmatrix} \qquad (3.21)$$

The relative velocity vector at point p expressed in global coordinate system is given by

$$\bar{v}_r^{(12)} = \dot{\phi} \begin{bmatrix} (-\sin\phi \cos\Psi) & -\cos\phi & -(\sin\phi \sin\Psi) & 0 \\ (\cos\phi \cos\Psi) & -\sin\phi & (\cos\phi \sin\Psi) & 0 \\ 0 & 0 & 0 & H \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ r\cos\theta \\ r\sin\theta \\ 1 \end{bmatrix} \quad \ldots (3.22)$$

At point of conjugacy,

$$\bar{n} \cdot \bar{v}_r^{(12)} = 0 \qquad (3.23)$$

From (3.21), (3.22) and (3.23) we get,

$$\theta = \tan^{-1} (- u/H) \text{ or } \pi + \tan^{-1} (- u/H) \qquad (3.24)$$

From (3.19) and (3.20) we get,

$$\begin{aligned} x &= u \cos\phi \cos\Psi - r\cos\theta \sin\phi + r\sin\theta \cos\phi \sin\Psi \\ y &= u \sin\phi \cos\Psi + r\cos\theta \cos\phi + r\sin\theta \sin\phi \sin\Psi \\ z &= -u \sin\Psi \qquad + r\sin\theta \cos\Psi + S \end{aligned} \quad \ldots. (3.25)$$

Equations (3.25) represent the oblique helicoid surface.

$$r_i \leq u \leq r_o$$

$$0 \leq \phi \leq 2\pi$$

3.3.3 Algorithm :

1. Read r_i, r_o, R, H, D, NDIV, NPHI, as input data.
2. Compute u_i $i = 1$, NDIV $r_i \leq u_i \leq r_o$.
3. Compute ϕ_K $K = 1$, NPHI $0 \leq \phi \leq 2\pi$
4. Compute $S_K = \phi_K H$ $K = 1$, NPHI

5. For a fixed value of ϕ_K, compute x,y,z coordinates using eq (3.24) and (3.25), varying u_i $i = 1,NDIV$.
6. Repeat the step 5 for K = 1, NPHI
7. Generate the required oblique helicoid with the NDIV X NPHI number of x, y, z coordinates computed.
8. Display front view, top view and isometric view of the surface.

3.3.4 Uses of Oblique Helicoid :

Two oblique helicoids partly form V-type screw thread.

Chapter 4

COMPUTATIONAL CONSIDERATIONS

4.1 Multiview Display Arrangement :

4.1.1 Device Independent Coordinates :

Different graphics devices have different screen sizes and different screen resolutions, the Tektronix 4006 display terminal screen has a maximum rectangular size of approximately 10 cms by 7.5 cms (1024 by 768 addressable points). For the Tektronix display surface in GPGS [13], device coordinates range from 0.0 to 1.3 horizontally and 0.0 to 1.0 vertically (and 0.0 to 1.0 in the z direction in case of 3D graphics device).

4.1.2 Viewports :

A GPGS user may specify a rectangular portion of the display surface within which the users picture is to be displayed. This rectangular region is called a viewport. A viewport is specified interms of device independent coordinates. The user can divide his display surface so that one part of display will not overlap the another part. We, Engineers can divide the space into four parts for displaying

front view, top view and isometric view separately in each display surface. We can use 2D or 3D viewports according to our use.

4.1.3 Windows :

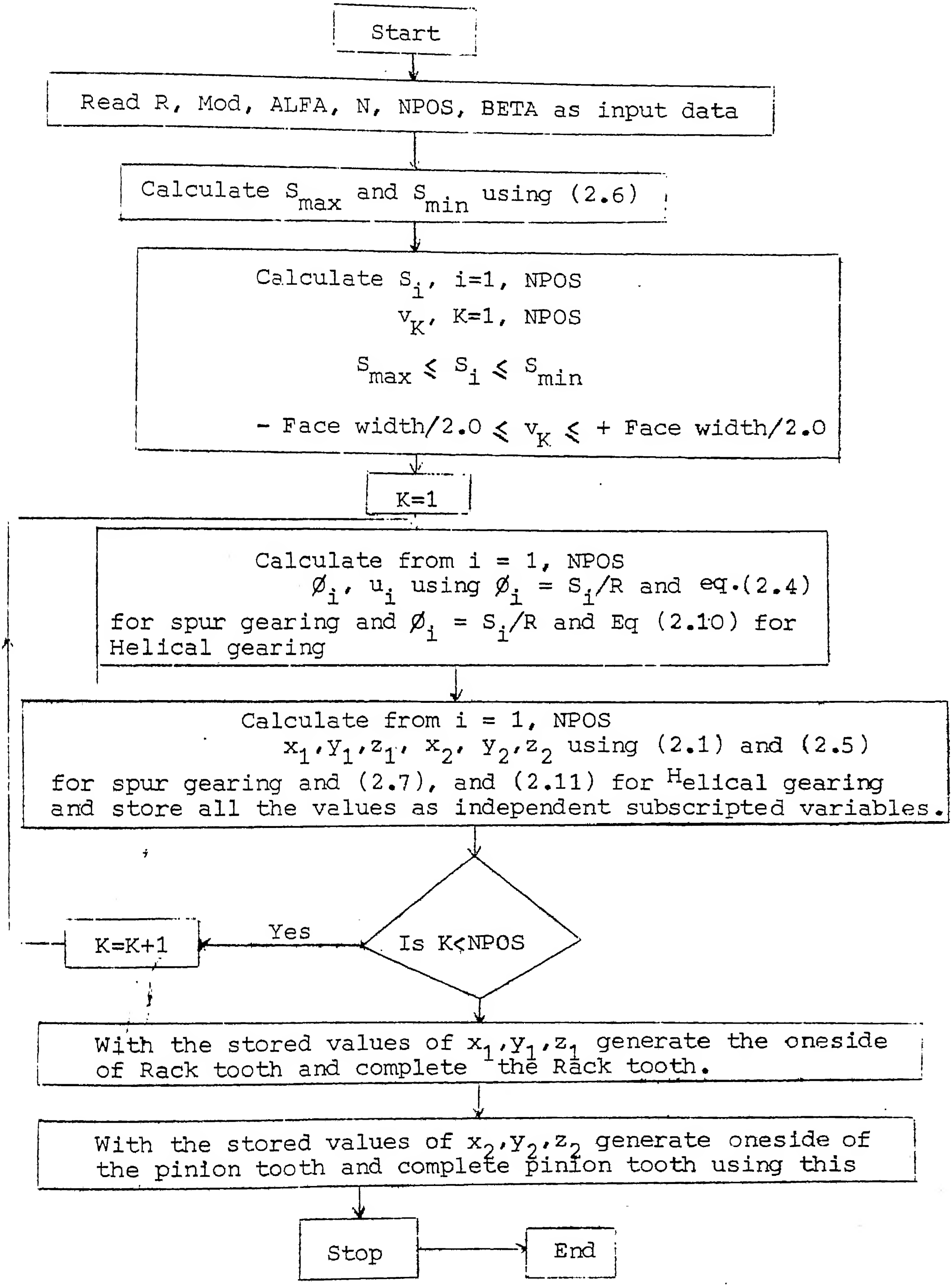
The natural coordinate system of the user's problem area is called as the 'user space' in GPGS. A GPGS user can specify a rectangular region in the user space as the region of immediate interest. This region is known as window. The window is mapped on to the viewport. GPGS users should see that their windows and respective viewports have the same proportions otherwise the mapping process will produce a unwanted scaling effects.

4.2 Surface Generation on the Screen:

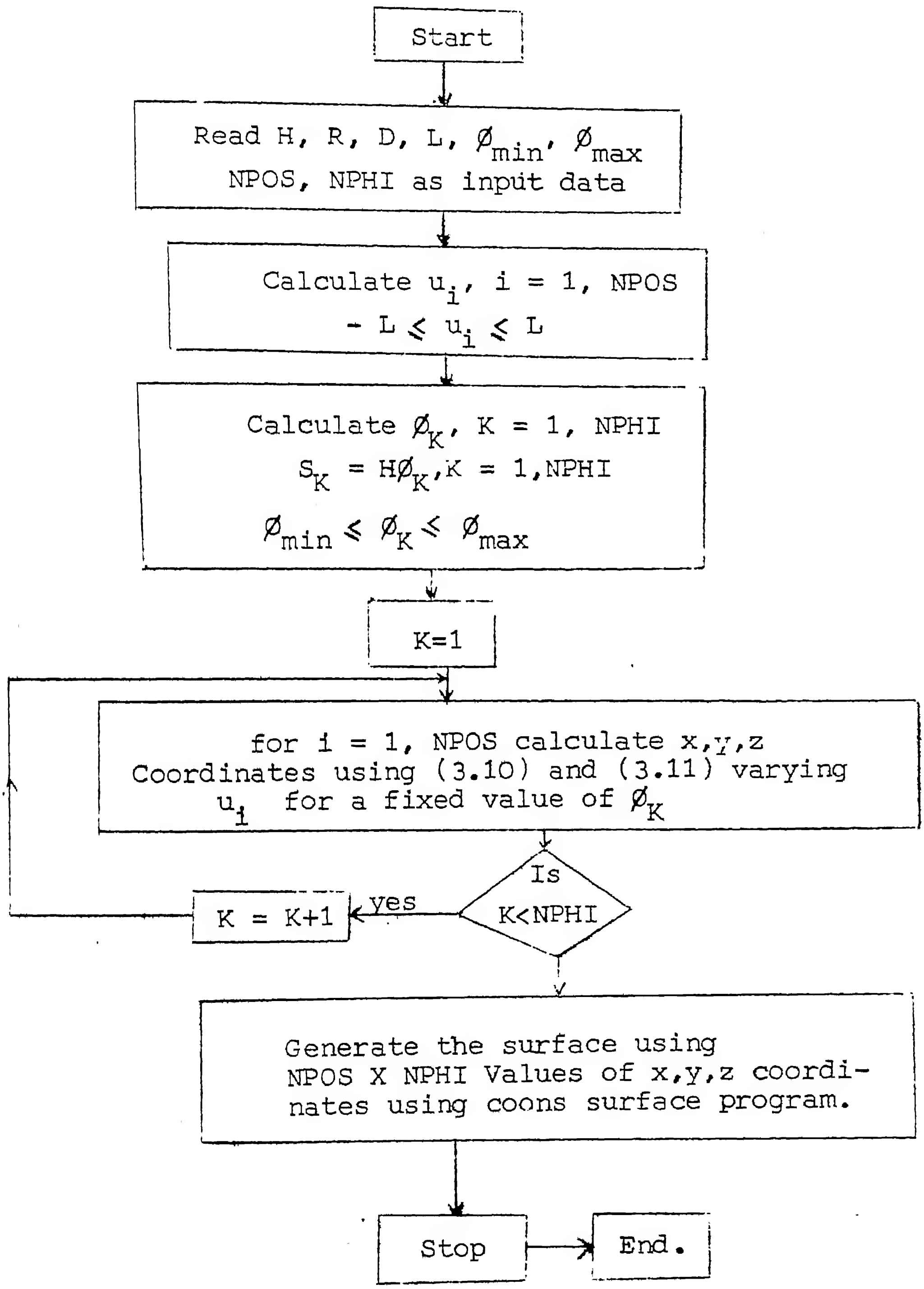
To develop the surface on the screen we have to give the points lying on the surface which are not coincident. The program for surface generation is developed based on the procedure given in [14] and [15]. This is coons surface patch generation in which the surface will be generated in terms of patches. The program does not work for the coincident points on the surface. The details about the coons surface generation is given in [15].

4.3 Flowcharts:

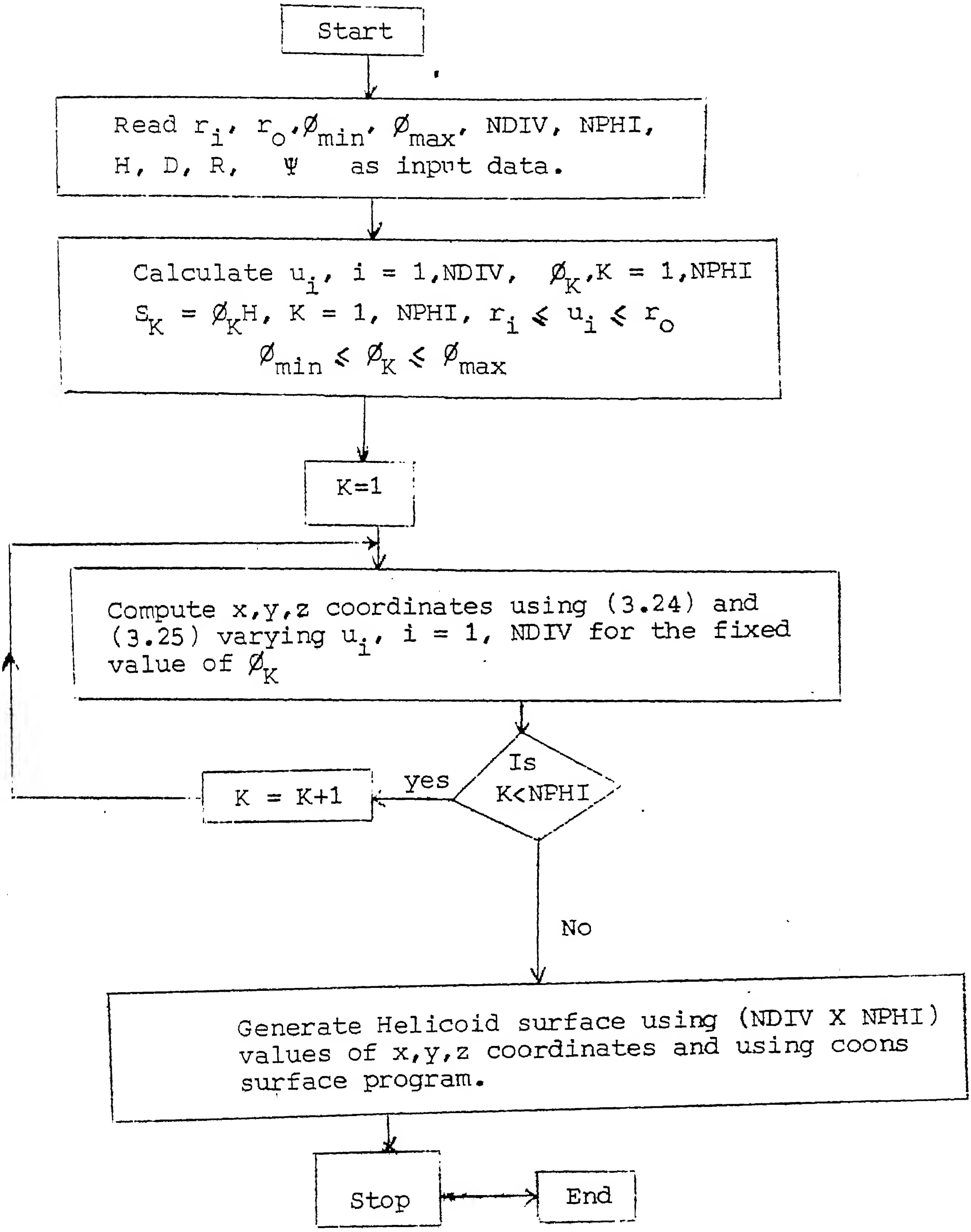
4.3.1 Surfaces in Spur and Helical Gearing :



4.3.2 Hyperbolic Paraboloid Surface :



4.3.3 Oblique Helicoid and Right Helicoid:



If Ψ = 0.0 Right helicoid will be generated, otherwise oblique helicoid will be generated.

4.4 Illustrative Examples :

4.4.1 Example 1 :

The input data used for the generation of surfaces in spur gearing involves pressure angle (ALFA), Number of teeth (N), Module of gear (MOD), Number of divisions (NPOS), Number of divisions in patch along u direction (NLU), Number of divisions in patch w direction (NLW). The Numerical data is given in Table 4.1 and the corresponding surface is given in Figure 4.1.

4.4.2 Example 2 :

The input data used for Helical gear surface generation includes pressure angle (ALFA), Helix angle (BETA), Number of teeth (N), Module of teeth (N), NPOS, NLU, NLW. The input data is given in Table 4.2 and the corresponding surface is given in Figure 4.2.

4.4.3 Example 3 :

The input data required for Hyperbolic paraboloid surface generation involves L, H, R, D, NPOS, NPHI, ϕ_{min}, ϕ_{max} with the terms explained in nomenclature. The input data is given in Table 4.3 and the corresponding surface is given in Fig. 4.3.

4.4.4 Example 4 :

The input data required for the generation of oblique Helicoid surface involves r_i, r_o, H, D, R, NDIV, NPHI, ϕ_{min}, ϕ_{max} and Ψ. The input data is given in Table 4.4 and the corresponding surface is given in Fig. 4.4.

4.4.5 Example 5 :

The input data is same as in Example 4 for right helicoid also but Ψ = 0.0 in the present case. The input data is given in Table 4.5 and the corresponding surface is given in Fig. 4.5.

TABLE 4.1

INPUT DATA FOR EXAMPLE 1

Number of teeth (N)	Module (MOD)	Pressure angle (ALFA)	No. of divisions (NPOS)	No. of divisions in patch along u direction (NLU)	along w direction (NLW)
20	5.00	20.00	11	3	3

TABLE 4.2

INPUT DATA FOR EXAMPLE 2

Number of teeth (N)	Module (MOD)	Pressure angle (ALFA)	Helix angle (BETA)	Number of divisions (NPOS)	No. of divisions angle u (NLU)	in patch along w (NLW)
20	5.00	20.00	15.00	11	3	3

TABLE 4.3

INPUT DATA FOR EXAMPLE 3

L	H	R	D	NPOS	NPHI	ϕ_{min}	ϕ_{max}
5.00	8.00	1.00	5.00	11	19	-45.00	45.00

TABLE 4.4

INPUT DATA FOR EXAMPLE 4

r_i	r_o	H	D	NDIV	NPHI	Ψ	ϕ_{min}	ϕ_{max}
5.00	15.00	8.00	5.00	11	37	30.00	0.00	360.00

TABLE 4.5

INPUT DATA FOR EXAMPLE 5

r_i	r_o	H	D	NDIV	NPHI	ϕ_{min}	ϕ_{max}
5.00	15.00	8.00	5.00	11	37	0.00	360.00

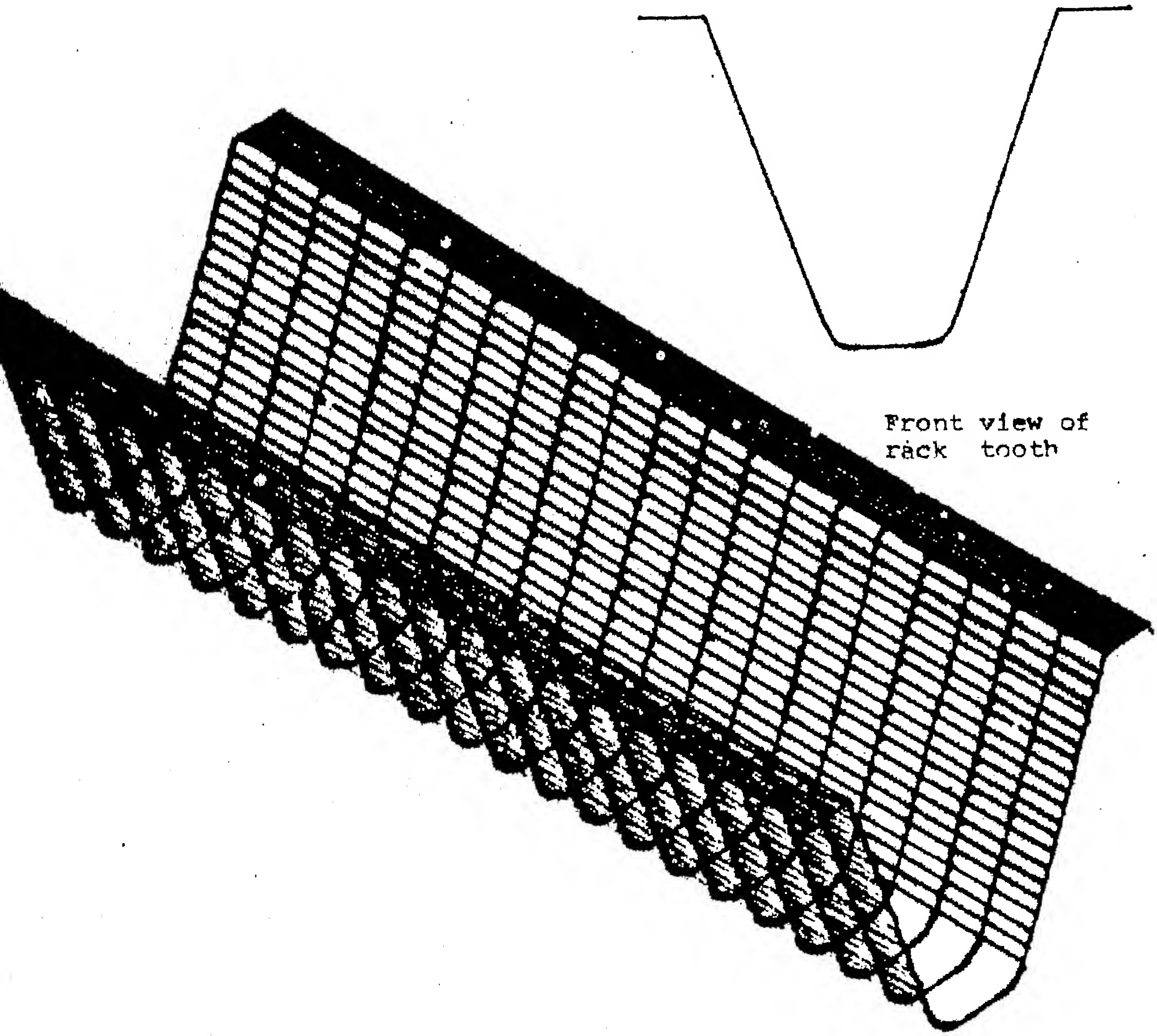


Fig. 4.1a Isometric view of a rack tooth

TOP VIEW OF RACK TOOTH

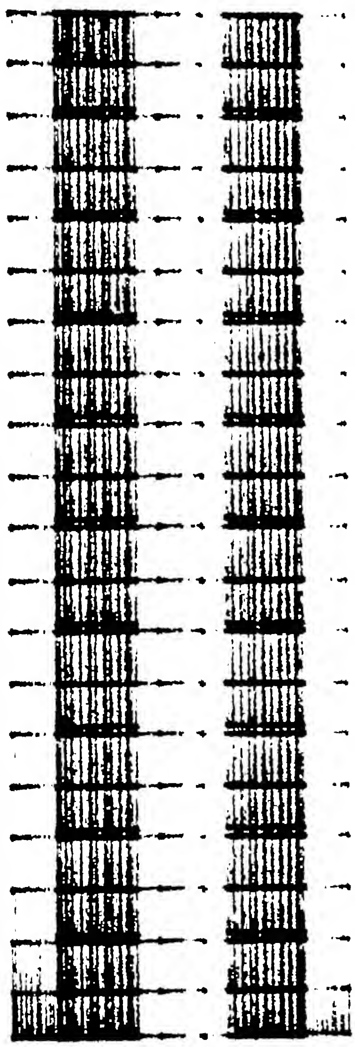


Fig. 4.1a

ISOMETRIC VIEW OF PINION TOOTH

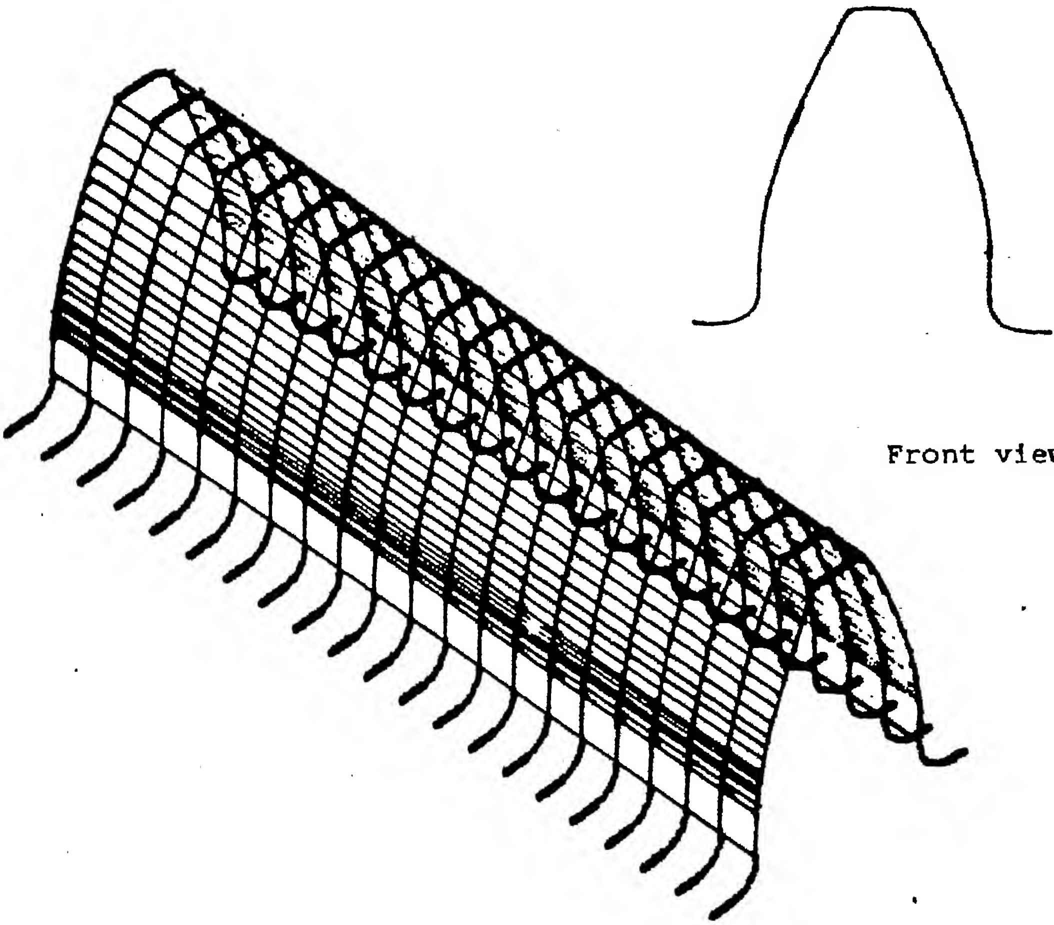


Fig. 4.1b

TOP VIEW OF PINION TOOTH

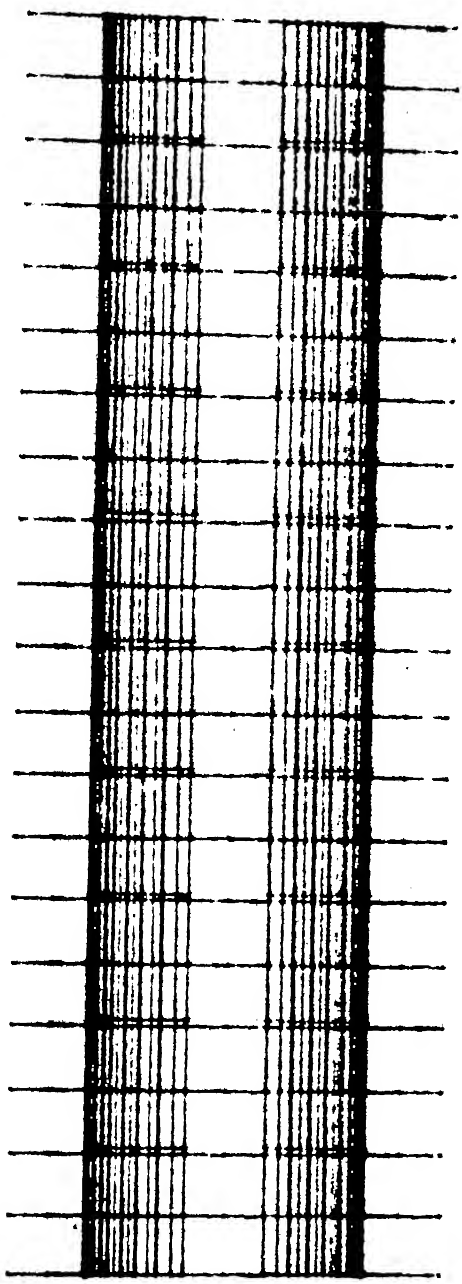


Fig. 4.1b

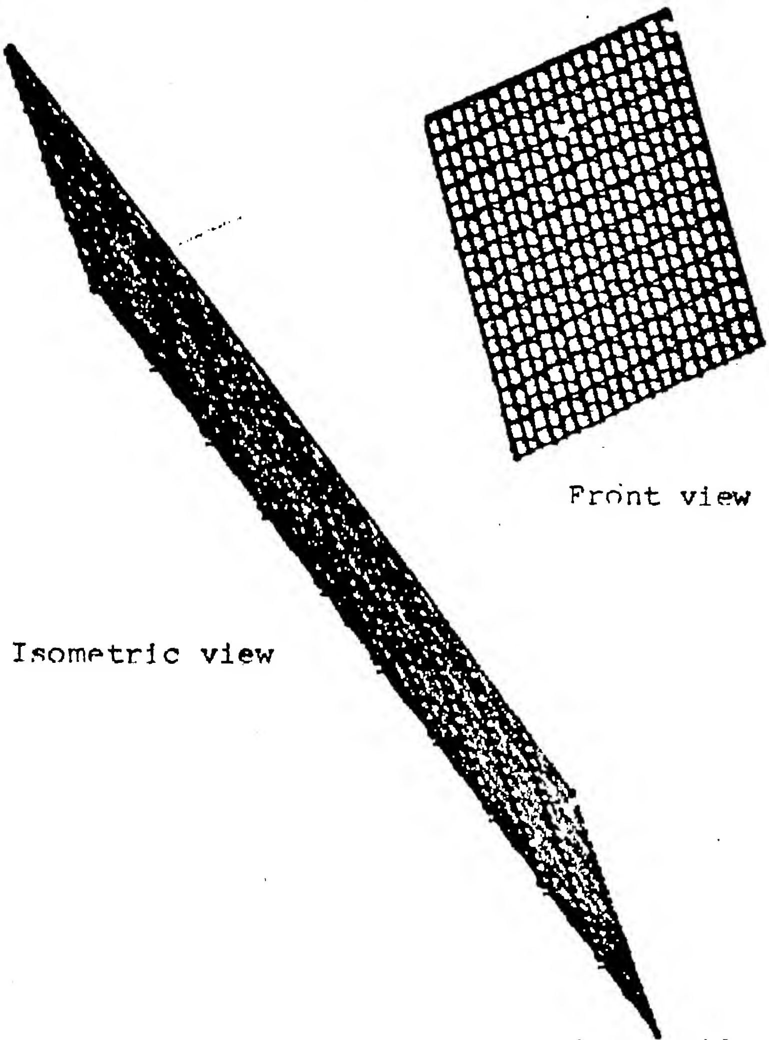


Fig. 4.2a Views of one side of helical rack surface

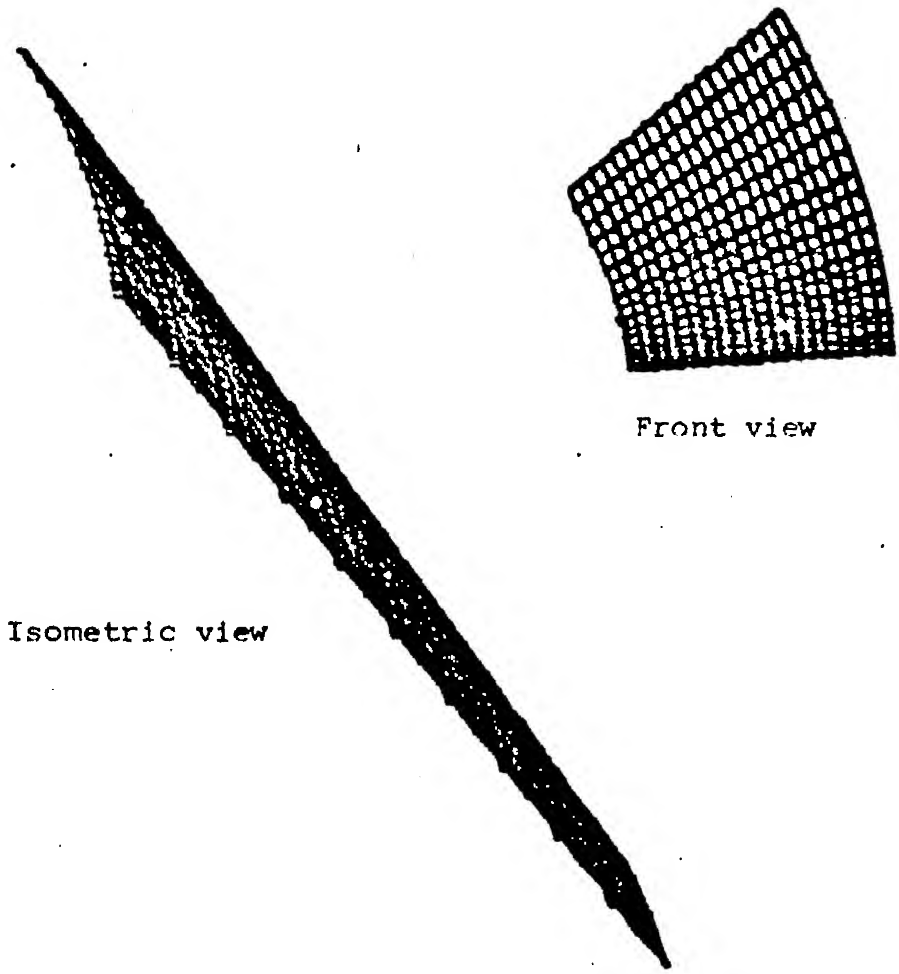


Fig. 4.2b Views of one side of helical pinion tooth surface

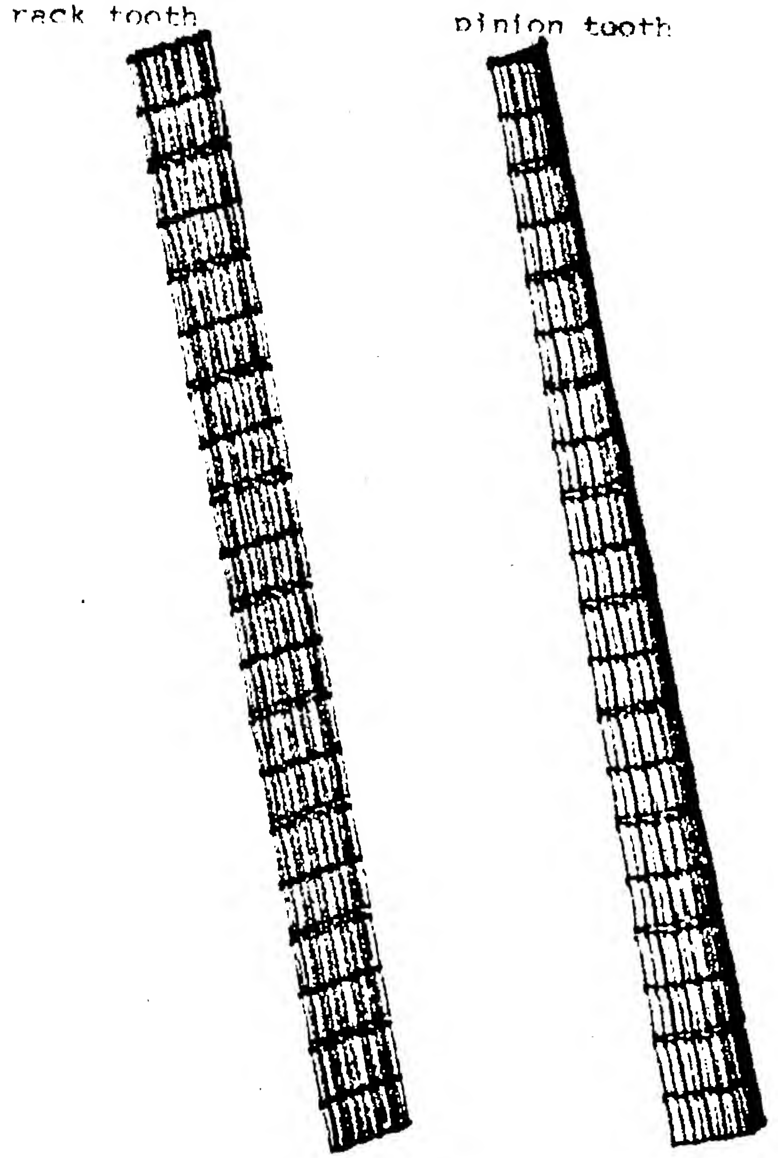


Fig. 4.2c Top views of rack and pinion (helical) tooth surfaces

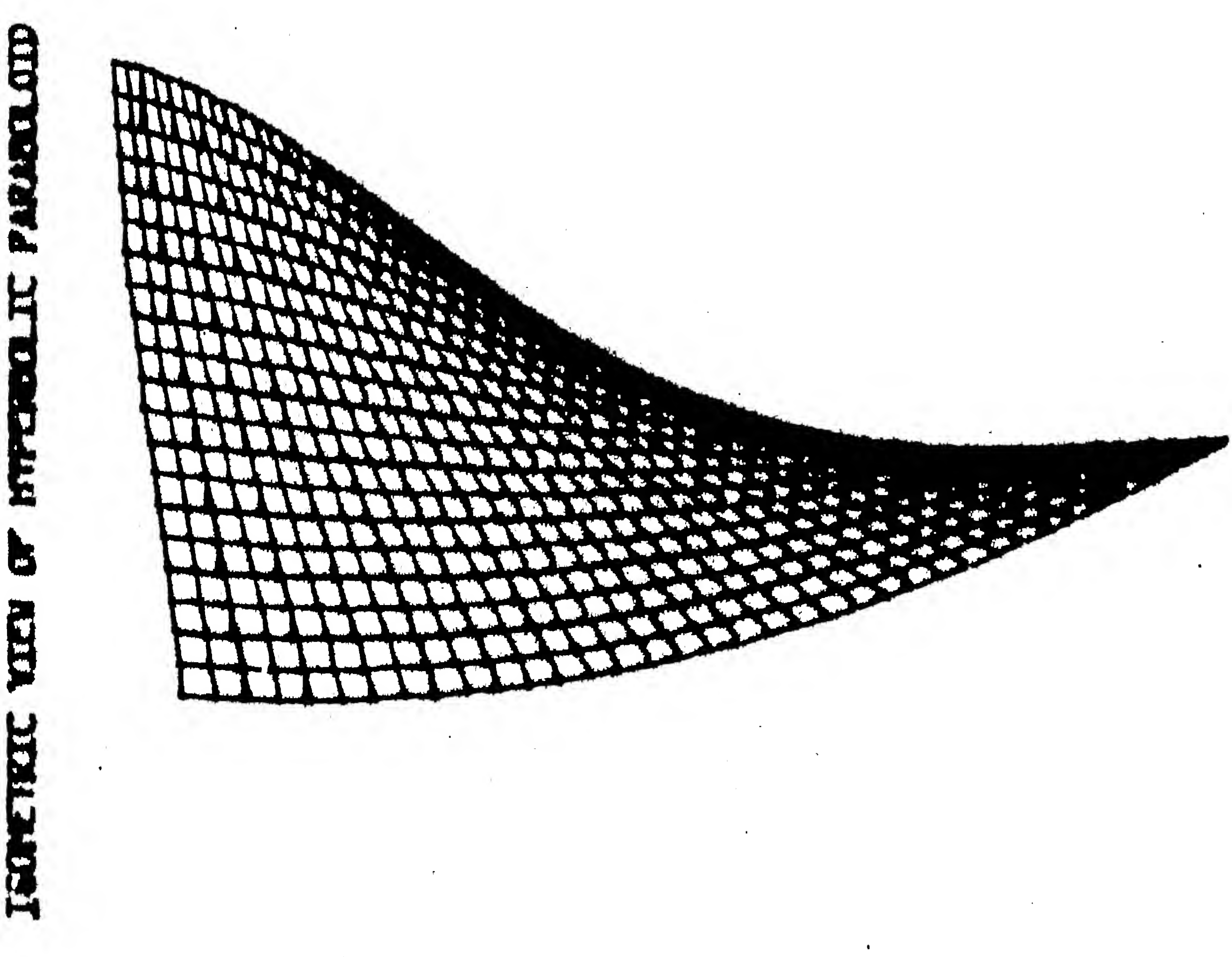
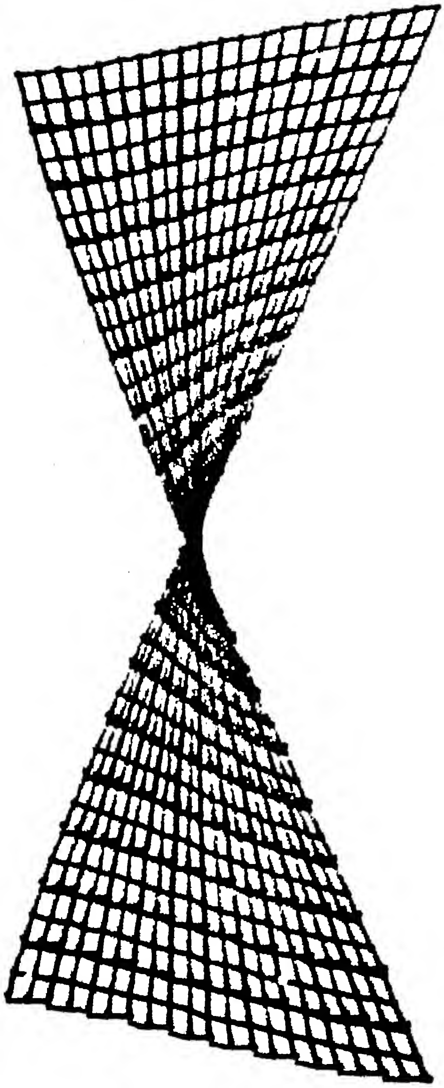
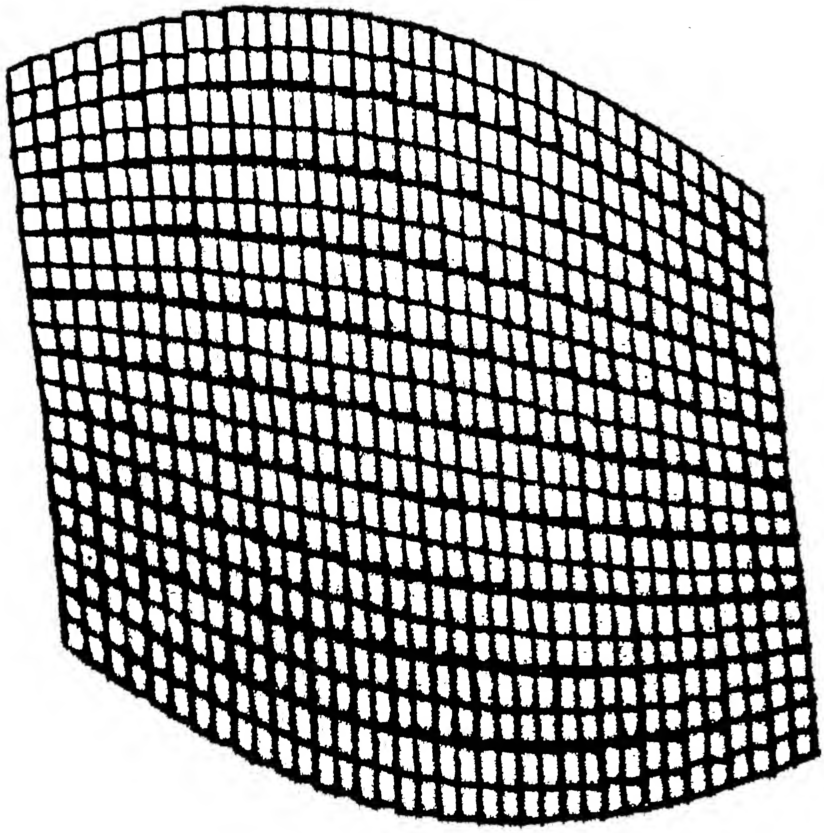


Fig. 4.3a



FRONT VIEW OF HYPERBOLIC PARABOLOID

Fig. 4.3b



TOP VIEW OF HYPERBOLIC PARABOLOID

Fig. 4.3c

ISOMETRIC VIEW OF OBLIQUE HELICOID

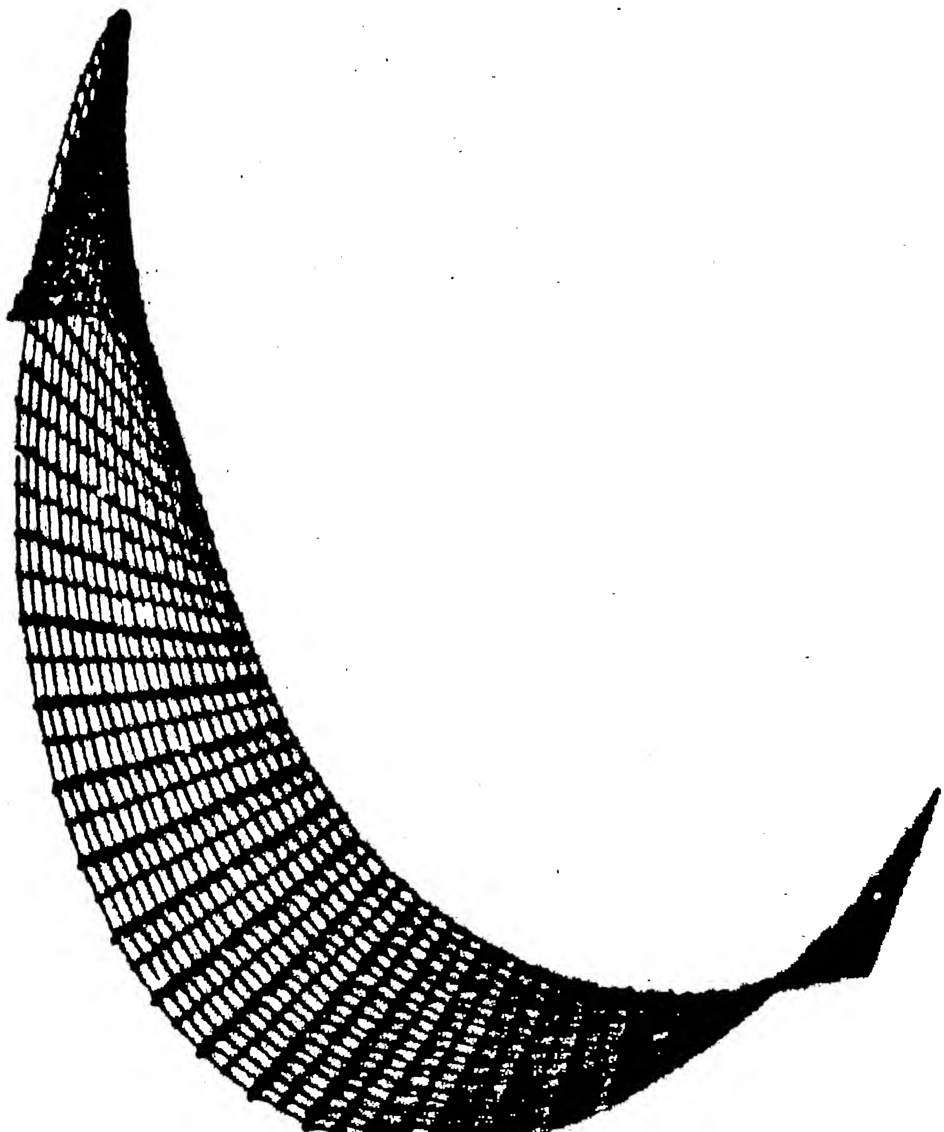


Fig. 4.4a

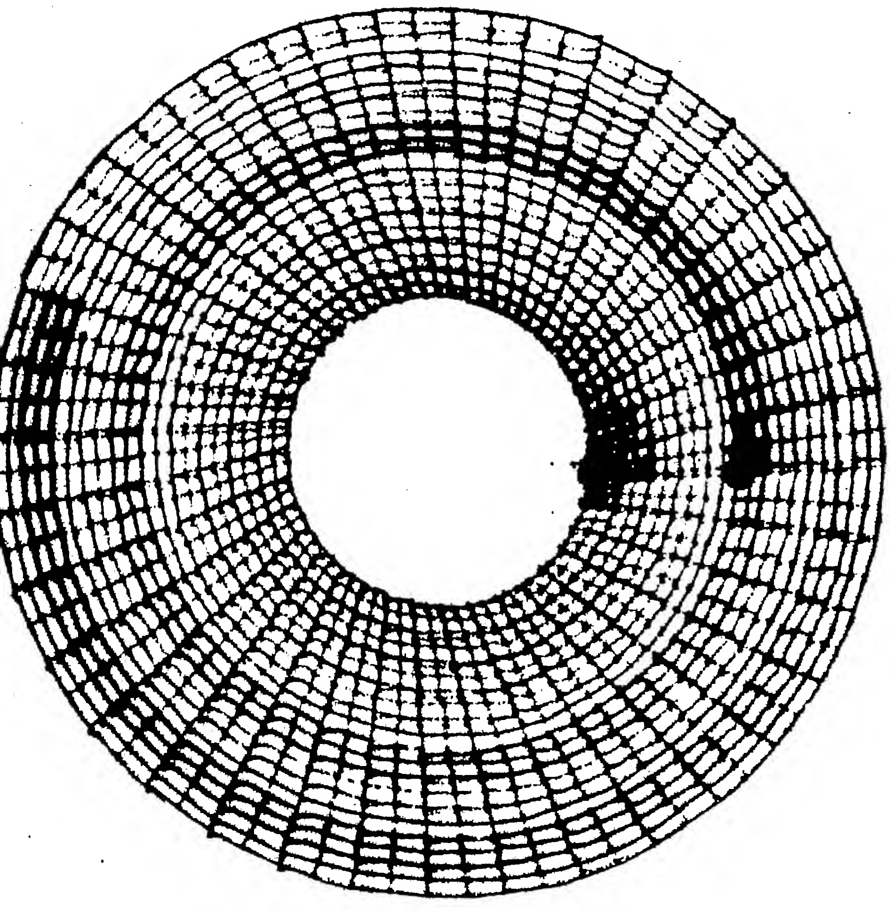


Fig. 4.4c

TOP VIEW OF OBLIQUE HELICOID.

ISOMETRIC VIEW OF RIGHT HELICOID.

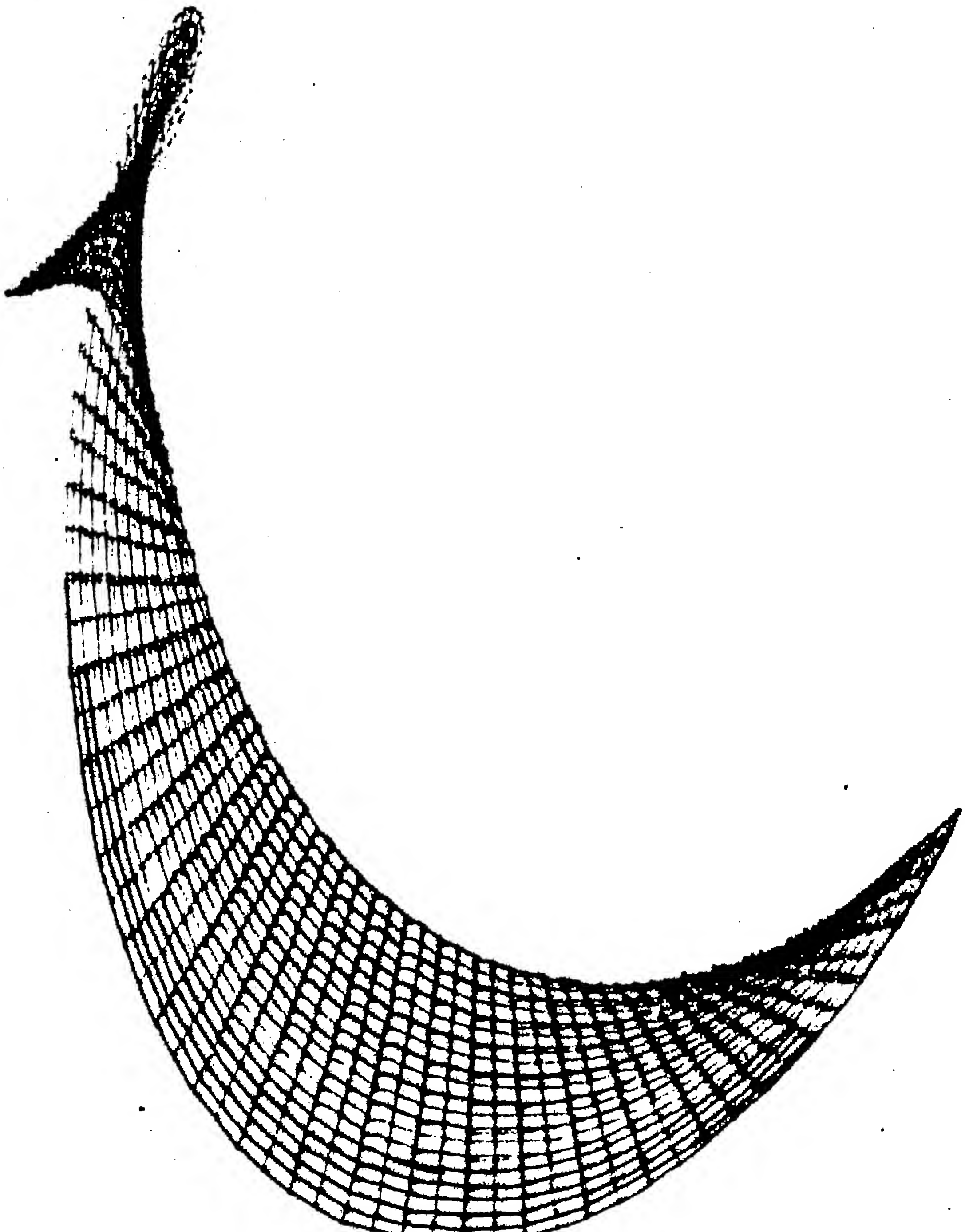


Fig. 4.5a

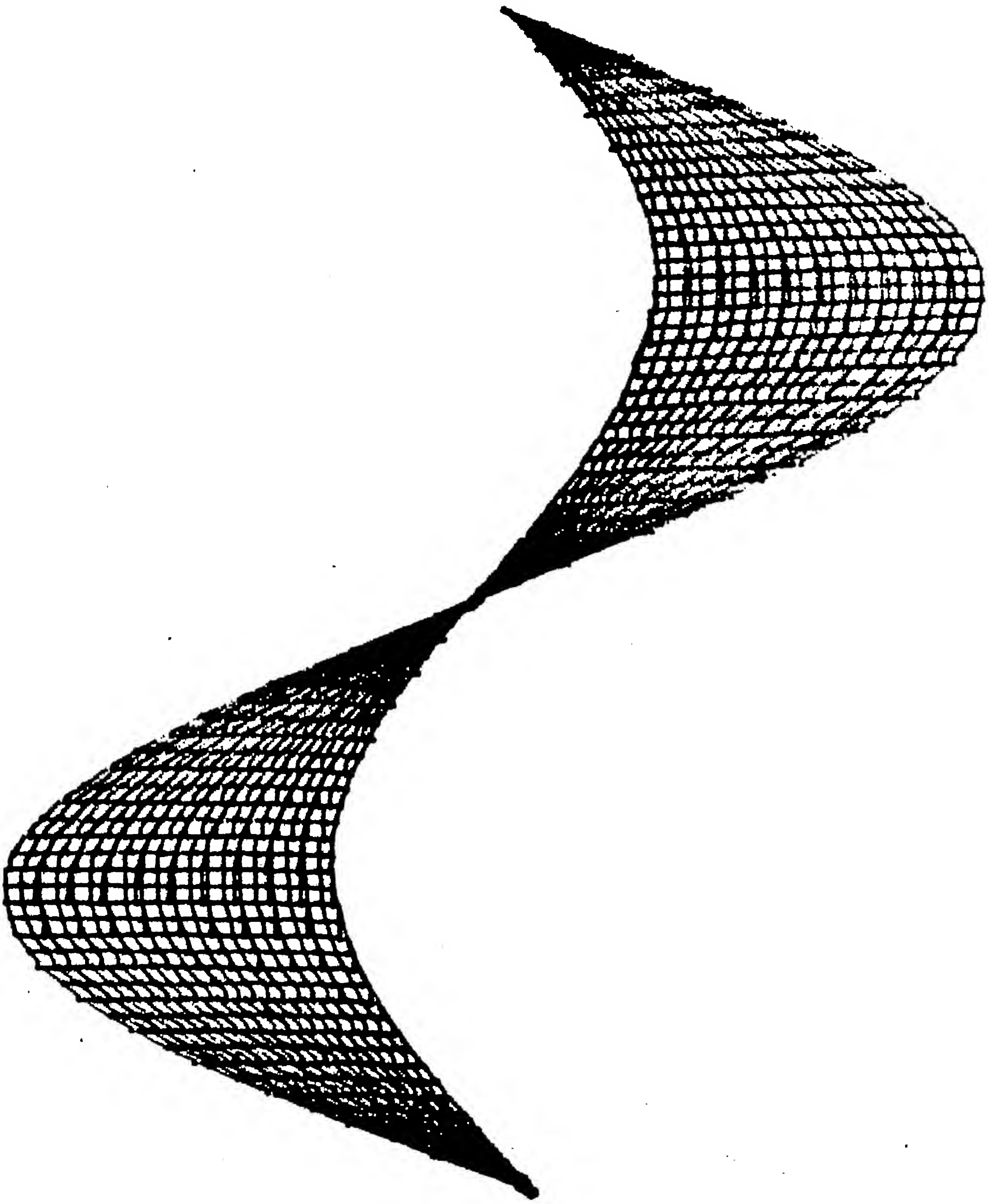
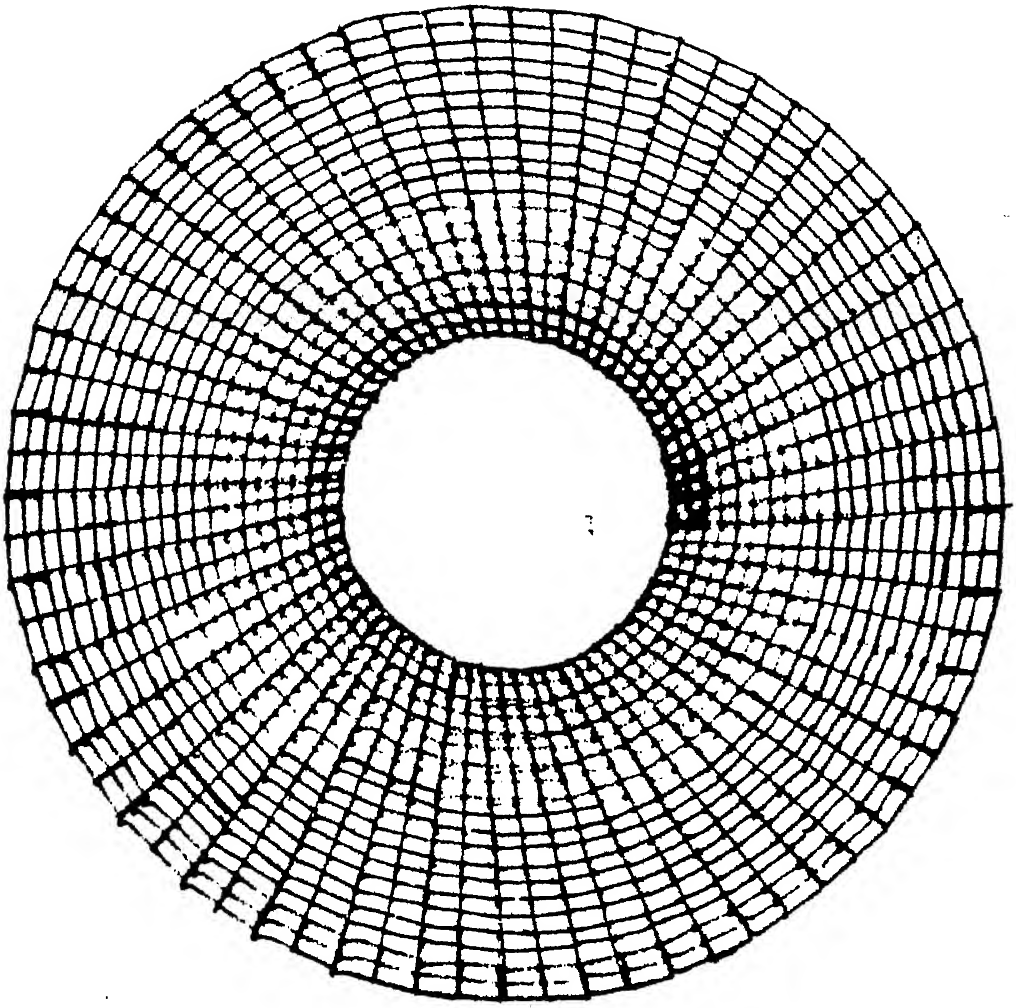


Fig. 4.5b Front view of right helicoid



TOP VIEW OF RIGHT HELICOID

Fig. 4.5c

Chapter 5

CONCLUSIONS

5.1 Technical Summary :

The present work deals with the generation and display of conjugate surfaces. To display conjugate surface one should develop a geometrical model which consists of formulation for the generation of the conjugate surface corresponding to a given surface. That is to say, given a surface model, the corresponding conjugate surface will be generated. In spur and helical gears, based on the rack profile (surface), the corresponding conjugate gear profile (surface) is generated. The geometrical models for the generation of conjugate gear profiles in spur and helical gears are discussed in Chapter 2. Similarly, the geometrical models for the generation of special conjugate surfaces are discussed in Chapter 3. As discussed in section 1.4, the hyperbolic paraboloid surface can be generated by milling machine with the special type of attachment which gives precession to the rotating cutter along its axis of rotation, the work piece should also move rectilinearly such that its velocity is related to the precession angular

velocity of the cutter by a constant. The program developed is in FORTRAN 10, GPGS and has been tested on DEC 1090 system. The program is general and one can generate the conjugate surfaces at his choice by changing the input data. Coons surface generation program is also general and one can generate any surface by feeding discrete points on the surface and the number of patches in the surface as the input data. This coons surface generation program is the integral part of the main program.

5.2 Recommendations for Future Work :

In helical gear tooth, the conjugate gear profiles are generated corresponding to the points on the rack profile. But one does not know if the points on the rack surface which will give the points on gear profile will lie within the boundaries of the (side of the) tooth face. Hence the tooth face can be developed by clipping or discarding the points on the gear profile which are lying outside the boundary of the tooth face. This is discussed in section 2.2.4.

In displaying the spur and helical gears, the curves which are behind one another are cluttered and the surface is not clear. One can remove this hidden curves using the hidden line algorithms [12] with slight modification. Since the curves are in side by side in the sequential

order, it is easy to eliminate the hidden curves using hidden line algorithms. This requires a lot of memory and CPU time.

Since the bevel gears, spiral gears, worm gears are of more practical use one can develop the formulation for the generation of these gears similar to that discussed in Chapter 2 and the interactive program can be developed to display these gears on the screen.

Spur and helical gears can be displayed by storing the generated single tooth and posting it at various instants to complete the full gear. This requires considerable memory. Special conjugate surfaces like cylindroid, conoid etc. can be displayed by developing the geometrical models similar to those discussed in Chapter 3. One can develop a interactive program by feeding the machine characteristics as input of the program developed for a particular machine.

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